Atomic Physics

Chapter 1

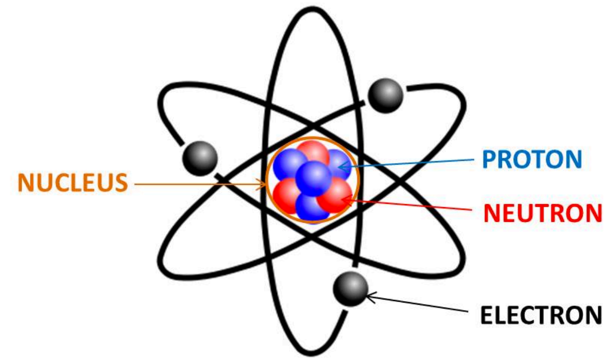
Basic Properties of Atom

What is an atom

An atom is the smallest unchangeable component of a chemical element.

1. Unchangeable means in this case by chemical means

2. Moderate temperatures: kT < eV



Mass range: 1.67×10^{-27} to 4.52×10^{-25} kg

Electric charge: zero (neutral), or ion charge

Diameter range: 62 pm (He) to 520 pm (Cs)

Components: Electrons and compact nucleus of protons and neutrons

The mass of an atom

Atomic mass unit (1u):

1/12 of the mass of a neutral carbon atom with nuclear charge 6 and mass number 12

Mass number (A):

The total number of protons and neutrons in nucleus

Avogadro constant (N_A):

1 mole of any substance contains the same number (N_A) of atoms (molecules)

$$N_{\mathrm{A}} = \frac{\text{Mass of 1 mole of the substance}}{\text{Mass of an atom}}$$
$$= 6.02214078(18) \times 10^{23}\ \mathrm{mol}^{-1}$$

Avogadro's number is a Bridge from macroscopic to microscopic physics.

1. The Faraday constant and elementary charge

$$F = N_{\mathrm{A}} e$$

2. Gas constant and Boltzmann constant

$$R = k_{\mathrm{B}} N_{\mathrm{A}}$$

3. Molar volume and atomic volume

$$V_{\mathrm{m}} = V_{\mathrm{atom}} N_{\mathrm{A}}$$

The mass of an atom

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The relation between 1u and N_A

$$1u = \frac{1}{N_A} = 1.660539040(20) \times 10^{-27}\ \mathrm{kg}$$

Electronvolt

$$\begin{aligned} 1\ \mathrm{eV} &= 1.602176565(35) \times 10^{-19}\ \mathrm{C} \times 1\ \mathrm{V} \\ &= 1.602176565(35) \times 10^{-19}\ \mathrm{J} \end{aligned}$$

Mass-energy equivalence

$$E = mc^2$$

1u transfer to eV

$$\begin{aligned} 1\ \mathrm{u} &= 931.478 \times 10^{6}\ \mathrm{eV}/c^2 \\ &= 931.478\ \mathrm{MeV}/c^2 \end{aligned}$$

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The mass of electron:

$$
\begin{aligned}
m_e &= 9.10938356(11) \times 10^{-31} \text{ kg} \\
&= 5.48579909070(16) \times 10^{-4} \text{ u} \\
&= 0.5109989461(31) \text{ MeV}
\end{aligned}
$$

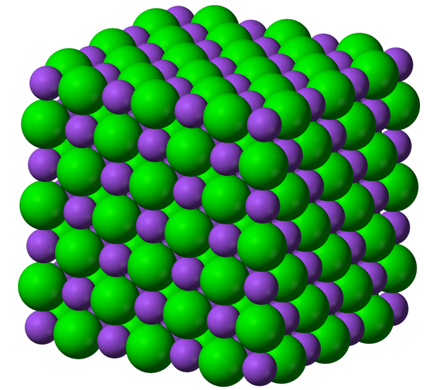
The mass of proton:

$$
\begin{aligned}
m_p &= 1.672621898(21) \times 10^{-27} \text{ kg} \\
&= 1.007276466879(91) \text{ u} \\
&= 938.2720813(58) \text{ MeV}
\end{aligned}
$$

The mass of neutron:

$$
\begin{aligned}
m_n &= 1.674927471(21) \times 10^{-27} \text{ kg} \\
&= 1.00866491588(49) \text{ u} \\
&= 939.5654133(58) \text{ MeV}
\end{aligned}
$$

Assume that the masses of 1 mole atoms is A, and the atom is spherical



$$\frac{4}{3}\pi r^3 N_{\rm A} = \frac{A}{\rho}$$

The radius of atom

The density of substance

The radius of atom

$$r = \left(\frac{3A}{4\pi\rho N_{\rm A}}\right)^{\frac{1}{3}}$$

The units for the radius of atom

$$1\ \text{nm} = 10^{-9}\ \text{m},\quad 1\ \text{Å} = 10^{-10}\ \text{m},$$

$$1\ \text{pm} = 10^{-12}\ \text{m},\quad 1\ \text{fm} = 10^{-15}\ \text{m}$$

The size of atom

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Elements	A	Density ρ (g/cm³)	Radius *r* (nm)
Li	7	0.7	
Al	27	2.7	
Cu	63	8.9	
S	32	2.07	
Pb	207	11.34	

The size of atom

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Li	7	0.7	0.16
Al	27	2.7	0.16
Cu	63	8.9	0.14
S	32	2.07	0.18
Pb	207	11.34	0.19

The size of atom

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The unit is pm

1	H 25																	He
2	Li 145	Be 105											B 85	C 70	N 65	O 60	F 50	Ne
3	Na 180	Mg 150											Al 125	Si 110	P 100	S 100	Cl 100	Ar
4	K 220	Ca 180	Sc 160	Ti 140	V 135	Cr 140	Mn 140	Fe 140	Co 135	Ni 135	Cu 135	Zn 135	Ga 130	Ge 125	As 115	Se 115	Br 115	Kr
5	Rb 235	Sr 200	Y 180	Zr 155	Nb 145	Mo 145	Tc 135	Ru 130	Rh 135	Pd 140	Ag 160	Cd 155	In 155	Sn 145	Sb 145	Te 140	I 140	Xe
6	Cs 260	Ba 215	*	Hf 155	Ta 145	W 135	Re 135	Os 130	Ir 135	Pt 135	Au 135	Hg 150	Tl 190	Pb 180	Bi 160	Po 190	At	Rn
7	Fr	Ra 215	**	Rf	Db	Sg	Bh	Hs	Mt	Ds	Rg	Cn	Nh	Fl	Mc	Lv	Ts	Og
Lanthanides	*	La 195	Ce 185	Pr 185	Nd 185	Pm 185	Sm 185	Eu 185	Gd 180	Tb 175	Dy 175	Ho 175	Er 175	Tm 175	Yb 175	Lu 175		
Actinides	**	Ac 195	Th 180	Pa 180	U 175	Np 175	Pu 175	Am 175	Cm	Bk	Cf	Es	Fm	Md	No	Lr		

Determination of the size of atom

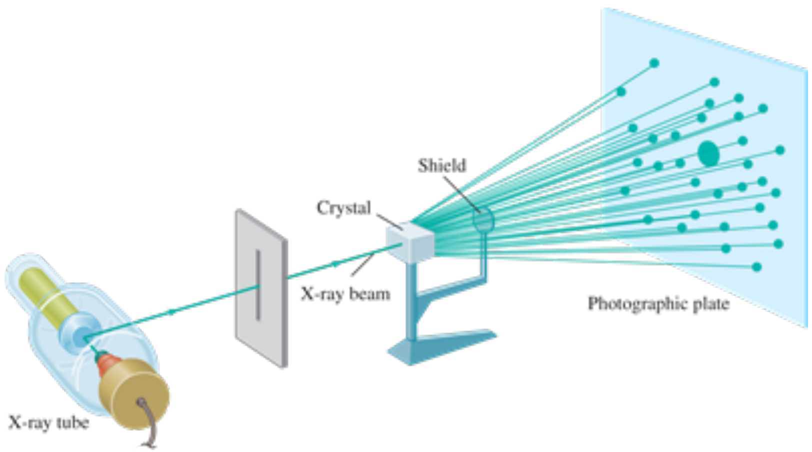
1. From the Covolume(协体积) in Van der Waals equation

$$(P + a/V^2)(V - b) = RT$$

where, the quantity b, is equal to the fourfold volume of the particles

$$b = 4\frac{4\pi}{3}r^3 N_{\mathrm{A}}$$

2. From X-ray diffraction measurements on crystals



3. From the interaction cross section

3. From the interaction cross section

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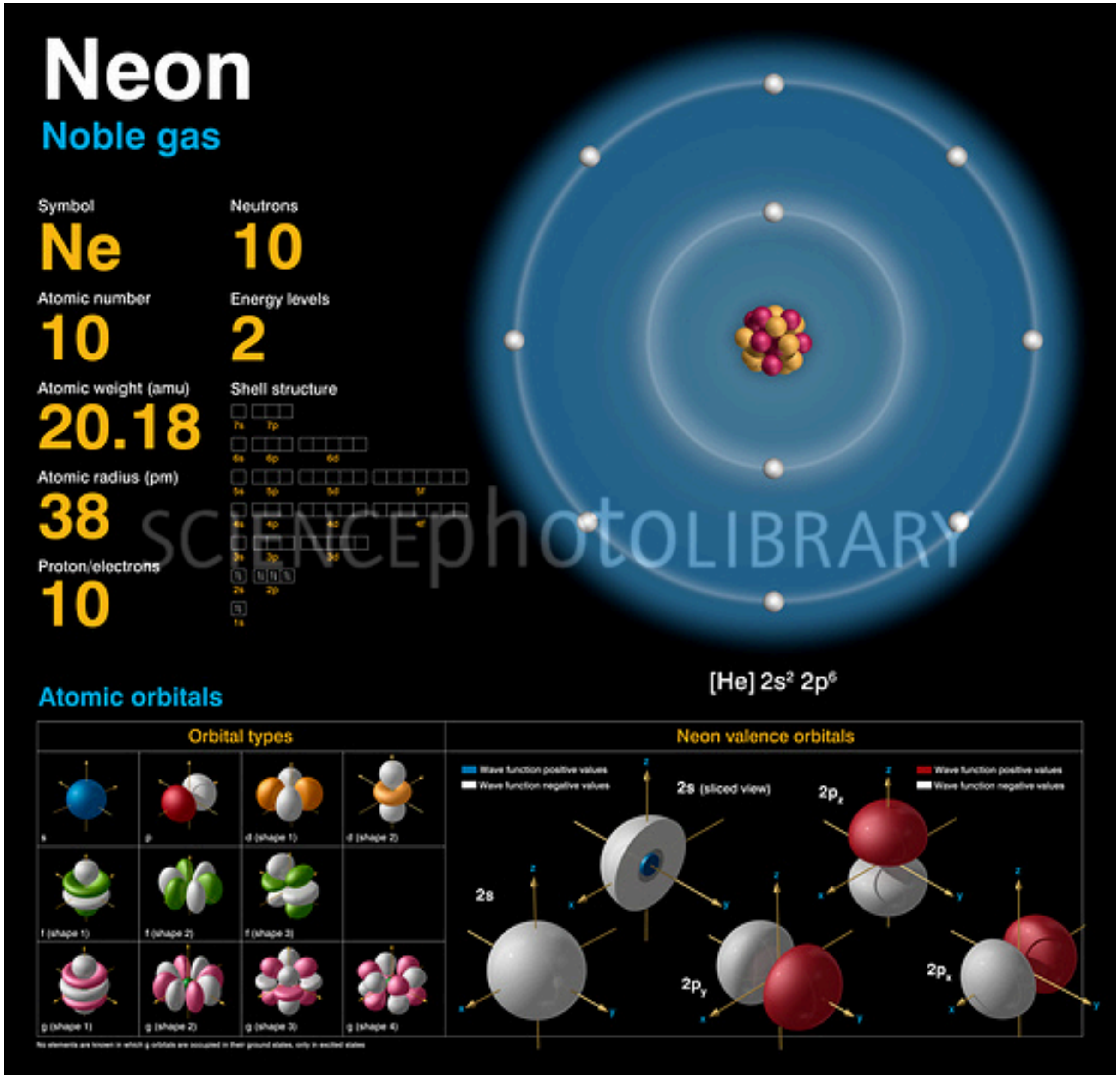
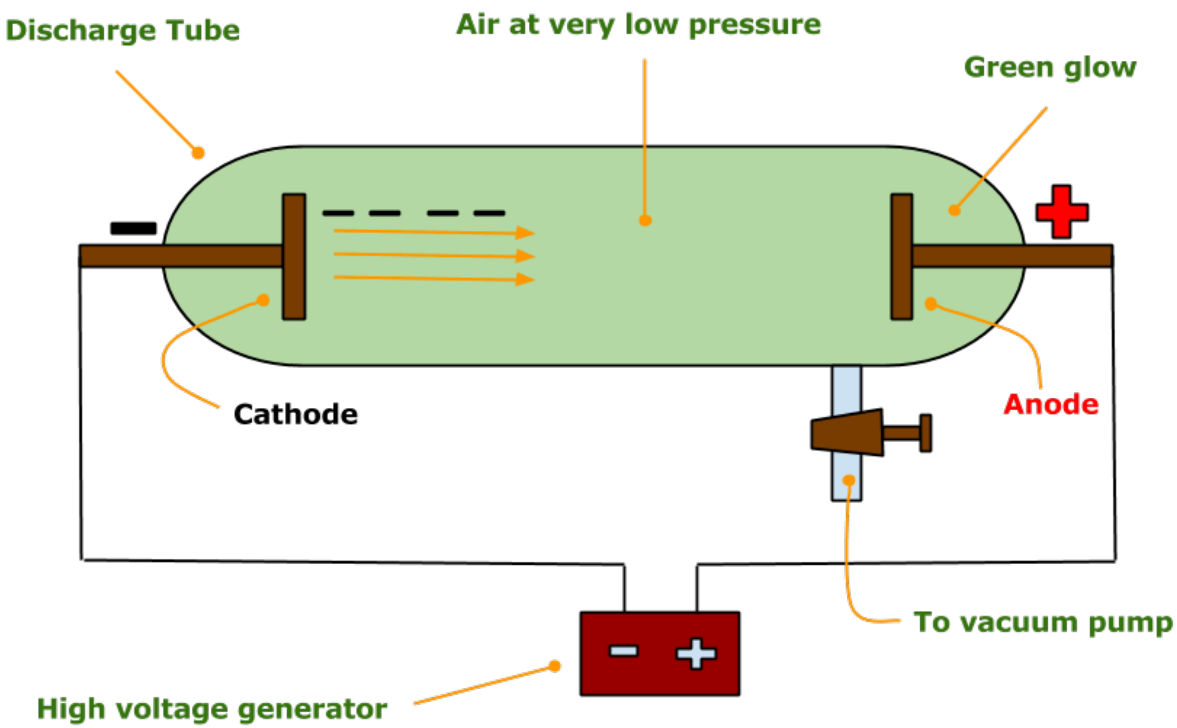
Element symbol

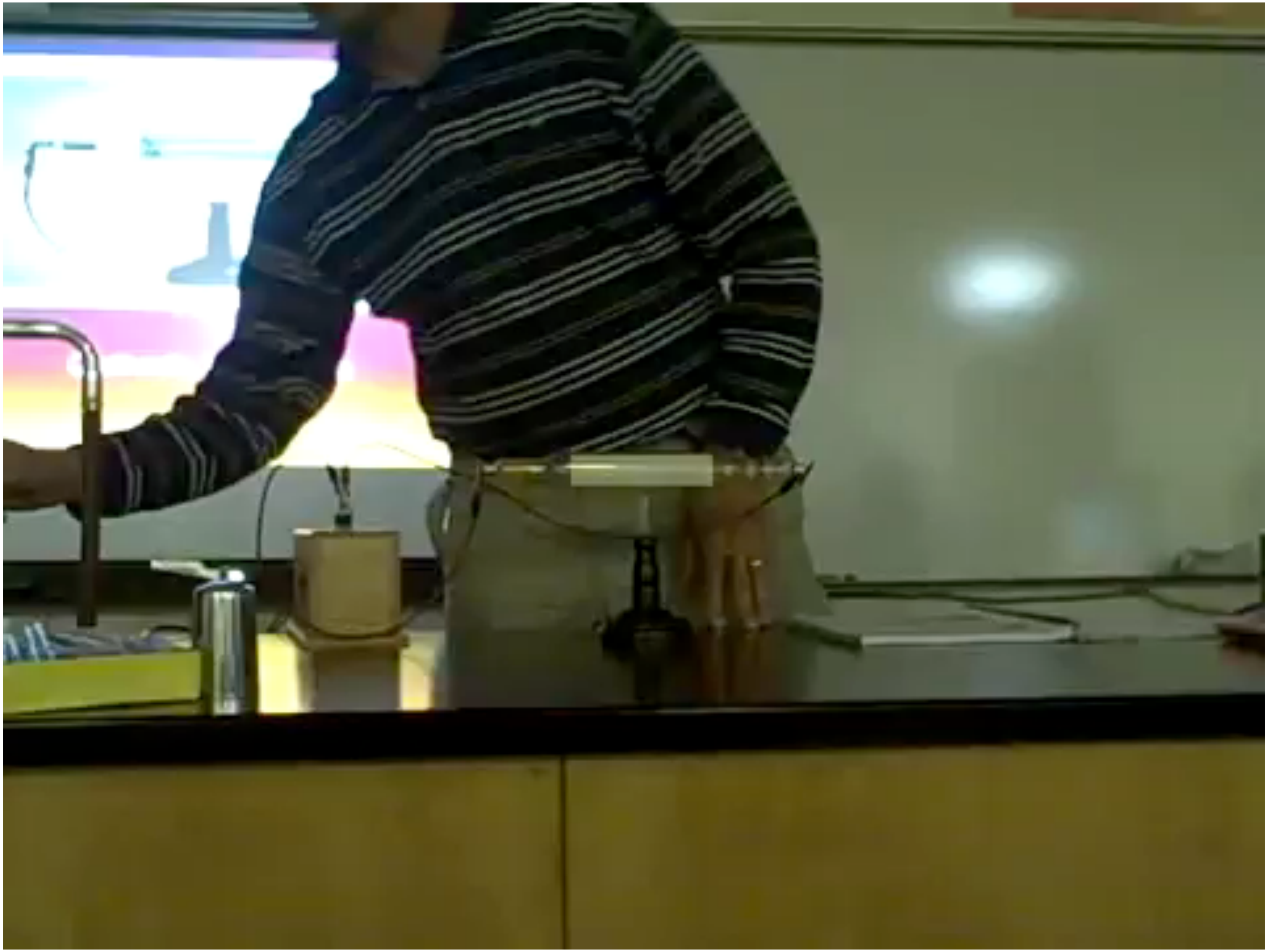


Cathode ray tube

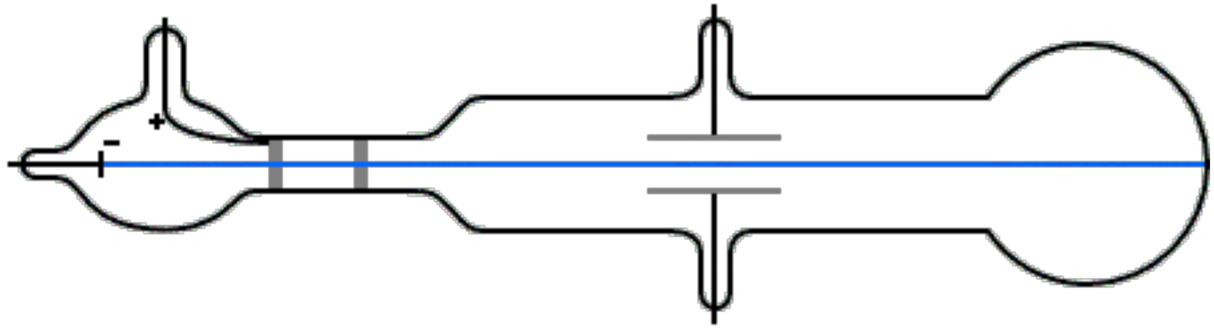


Production of cathode rays

Cathode ray tube



The discovery of electron



http://web.lemoyne.edu/~giunta/thomson1897.html

1. They travel in straight lines.
2. They are independent of the material composition of the cathode.
3. Applying electric field in the path of cathode ray deflects the ray towards positively charged plate. Hence cathode ray consists of negatively charged particles.

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Charge-to-Mass Ratio for the Electron

Gas.	Value of W/Q.	I.	m/e	v.
		Tube 1.		
Air	4.6×10^{11}	230	$.57\times10^{-7}$	4×10^{9}
Air	1.8×10^{12}	350	$.34\times10^{-7}$	1×10^{10}
Air	6.1×10^{11}	230	$.43\times10^{-7}$	5.4×10^{9}
Air	2.5×10^{12}	400	$.32\times10^{-7}$	1.2×10^{10}
Air	5.5×10^{11}	230	$.48\times10^{-7}$	4.8×10^{9}
Air	1×10^{12}	285	$.4\times10^{-7}$	7×10^{9}
Air	1×10^{12}	285	$.4\times10^{-7}$	7×10^{9}
Hydrogen	6×10^{12}	205	$.35\times10^{-7}$	6×10^{9}
Hydrogen	2.1×10^{12}	460	$.5\times10^{-7}$	9.2×10^{9}
Carbonic acid	8.4×10^{11}	260	$.4\times10^{-7}$	7.5×10^{9}
Carbonic acid	1.47×10^{12}	340	$.4\times10^{-7}$	8.5×10^{9}
Carbonic acid	3.0×10^{12}	480	$.39\times10^{-7}$	1.3×10^{10}

Contemporary theories of atom

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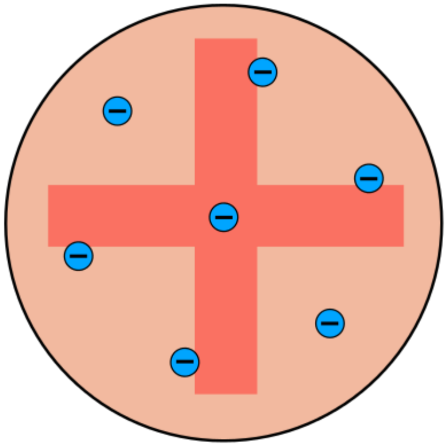
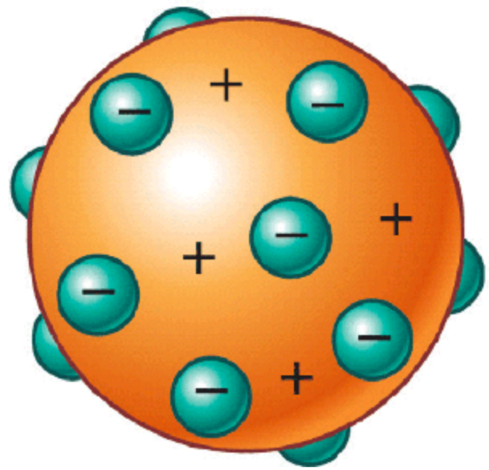
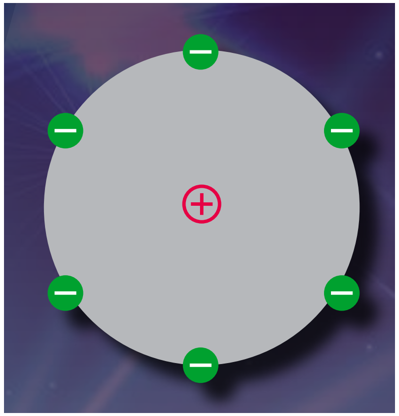
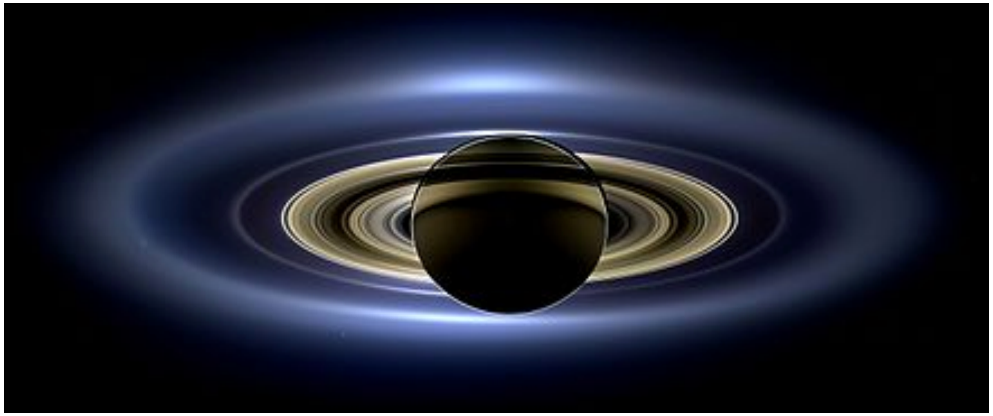
1. Plum pudding model (Lord Kelvin and J. J. Thomson)



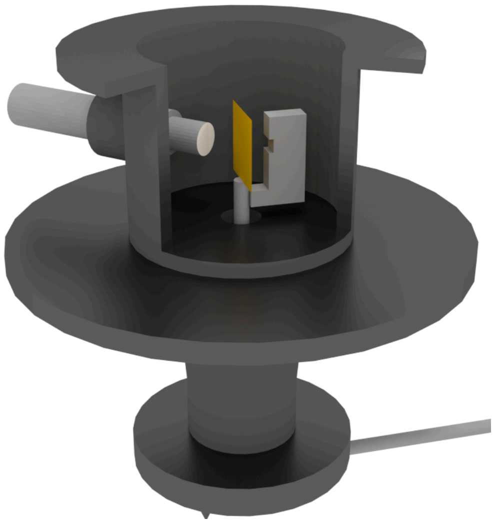


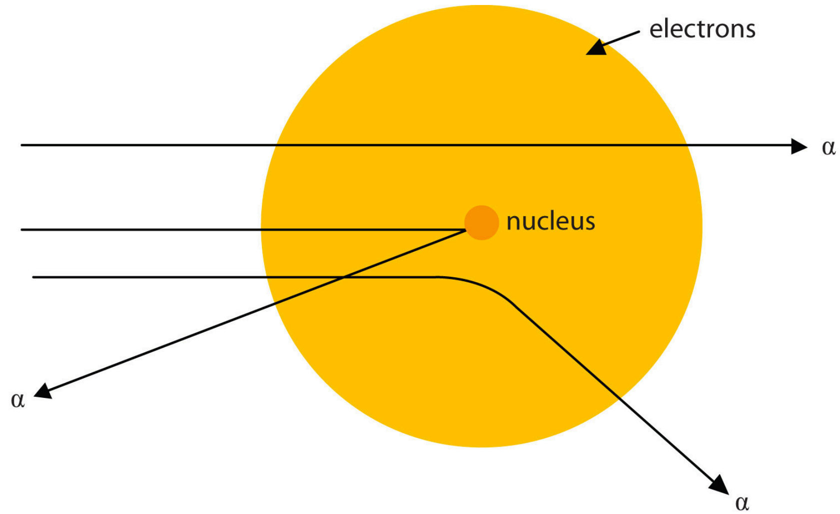
2. Saturnian model of the atom (H. Nagaoka)

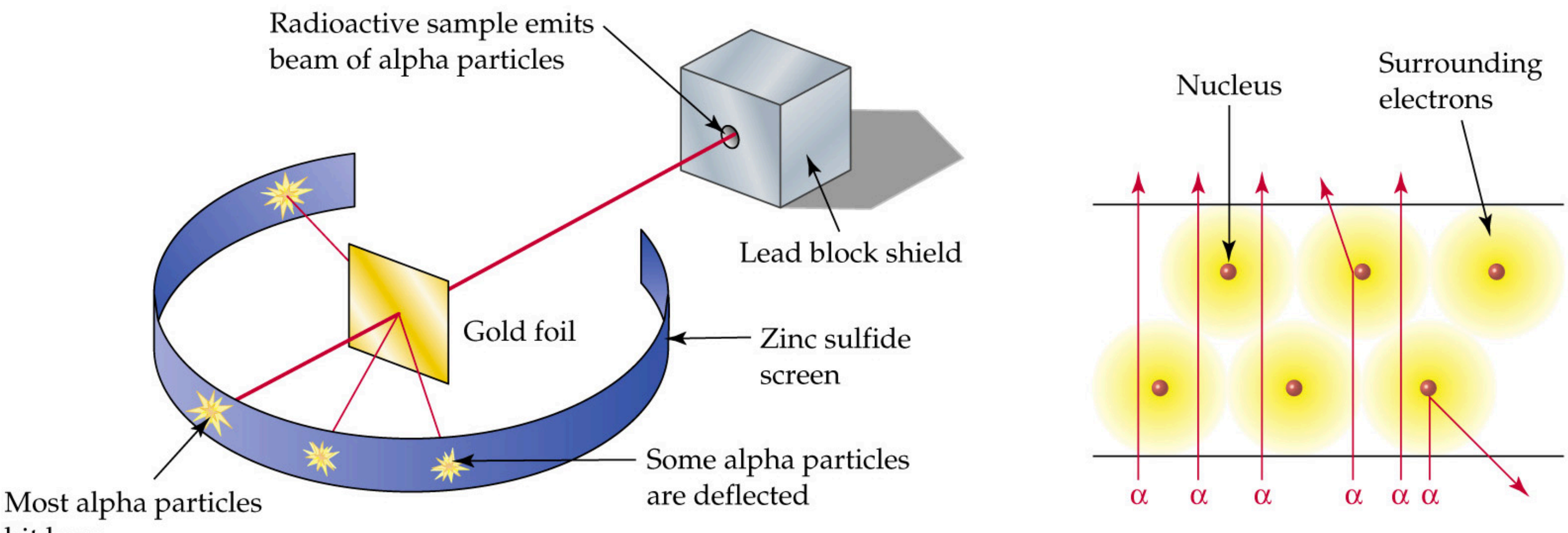




Geiger-Marsden experiment



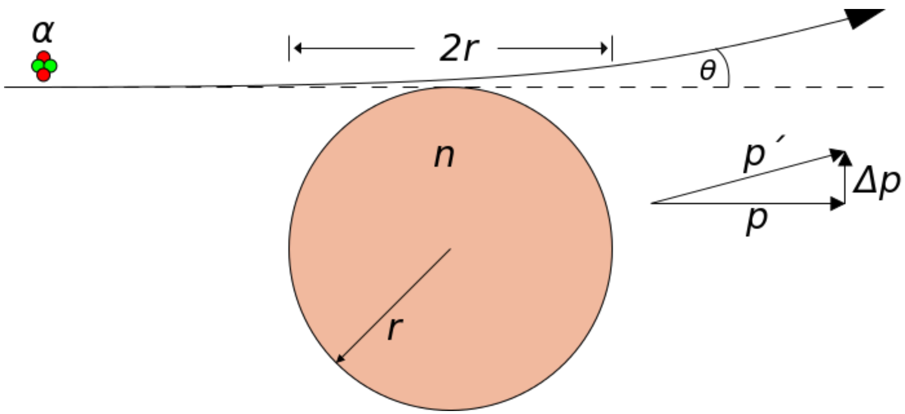




(a) (b)

Implications of plum pudding model

Using classical physics, the alpha particle's lateral change in momentum Δp can be approximated using the impulse of force relationship and the Coulomb force expression:



$$\Delta p = F\Delta t = k\frac{Q_\alpha Q_n}{r^2}\frac{2r}{v_\alpha}$$

The maximum deflection angle:

$$\theta \approx \frac{\Delta p}{p} < k\frac{2Q_\alpha Q_n}{m_\alpha r v_\alpha^2} = 0.000326 \text{ rad}$$

where, r: radius of a gold atom

Q_n: positive charge of gold atom

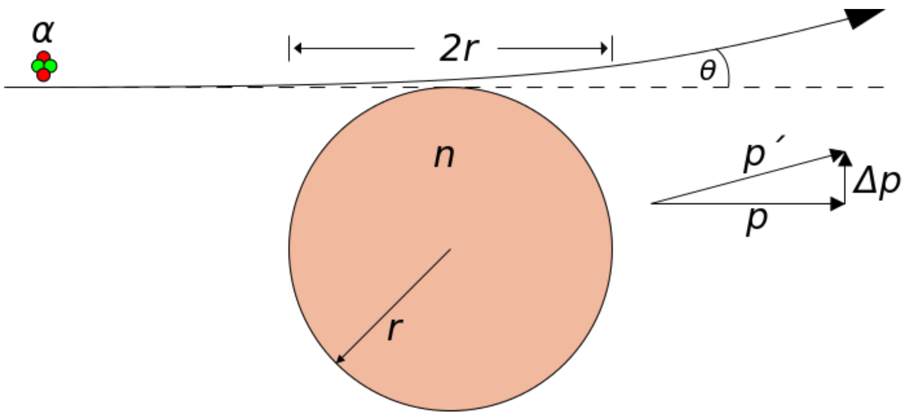
Q_α: charge of alpha particle.

k: Coulomb's constant

m_α: mass of alpha particle

v_α: velocity of alpha particle

Implications of plum pudding model



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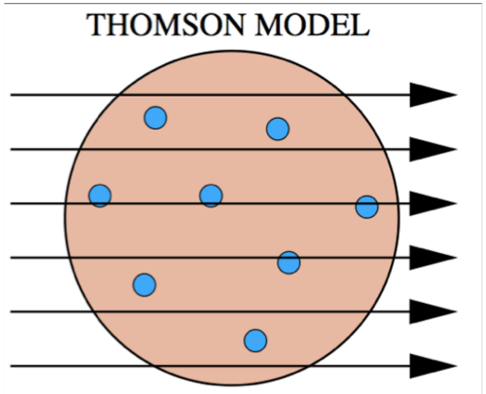

Fig – Thomson Lpum Pudding Model Source - Wikipedia

where, r: radius of a gold atom

Q_n: positive charge of gold atom

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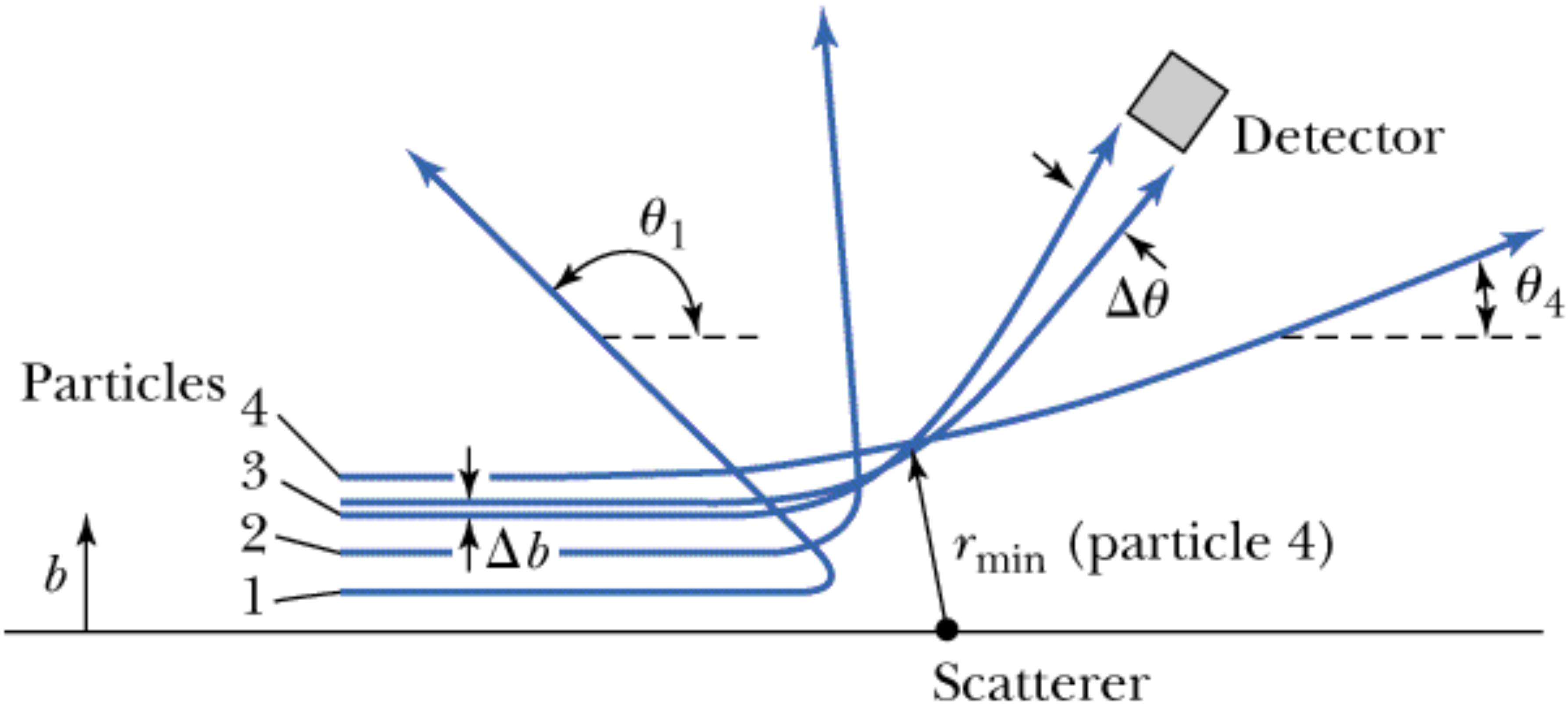
v_α: velocity of alpha particle

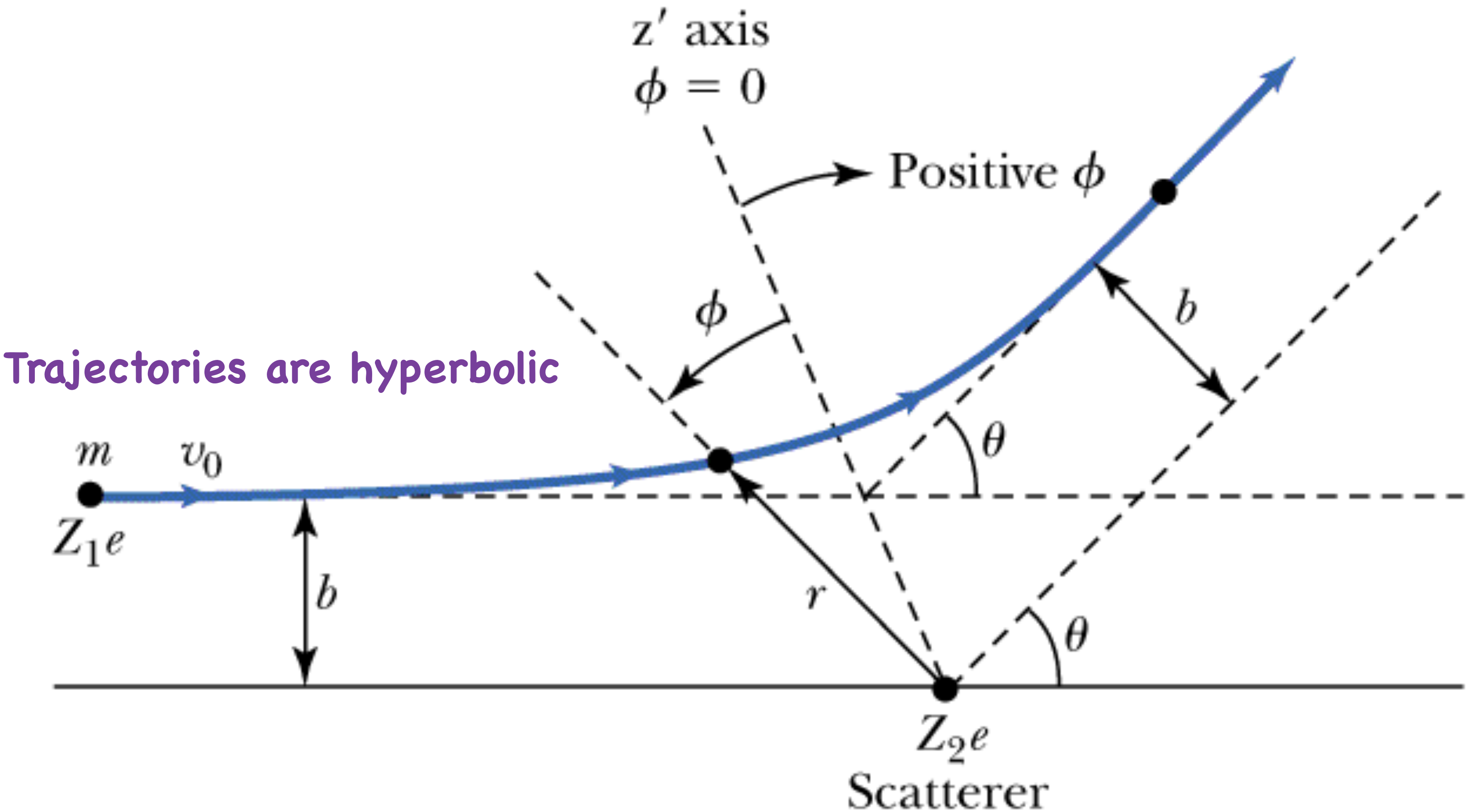
Trajectories in Scattering

Trajectories are strongly dependent on the impact parameter



Rutherford Scattering Formula

The key concept in Rutherford scattering is the relationship between the impact parameter b and the scattering angle θ.



1. Basic knowledge

The Coulomb force;

The Newton's laws;

The conservation of linear momentum;

The conservation of angular momentum.

2. Assumptions

Single scattering

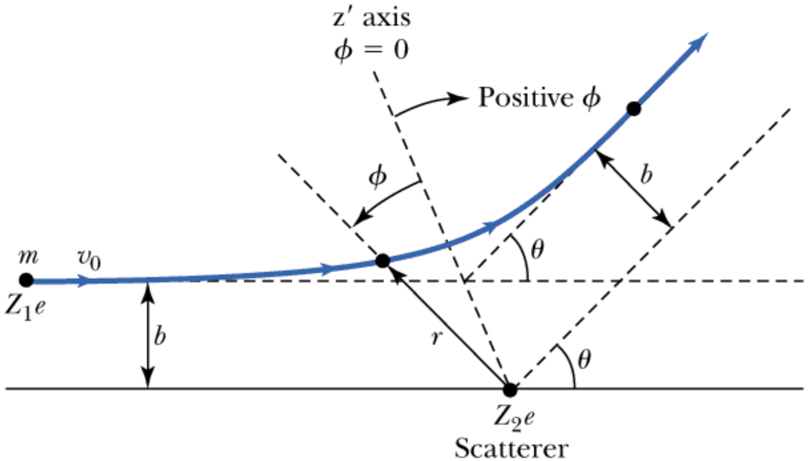
Only consideration Coulomb force

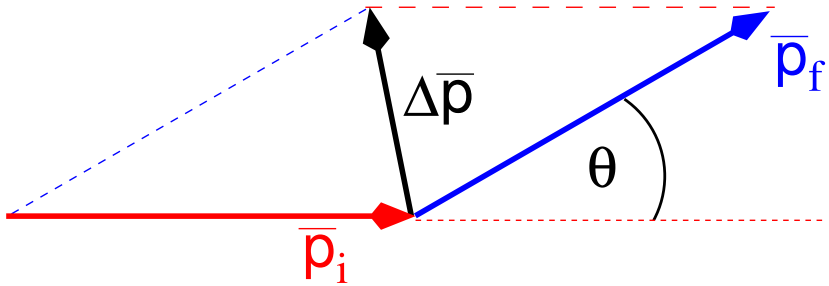
The effect of electrons in nuclei is neglected

The target is static

Rutherford Scattering Formula

Momentum change in Rutherford scattering



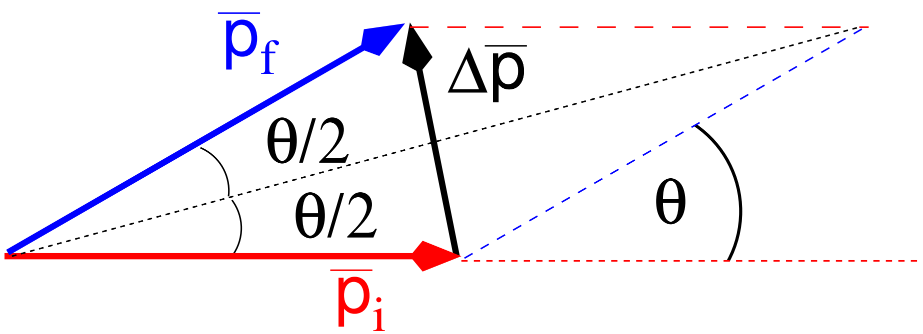


Elastic scattering

$$|\vec{p}_i| = |\vec{p}_f| = p$$

Momentum change

$$\Delta p = 2p\sin(\theta/2)$$



Rutherford Scattering Formula

From the Newton's second law

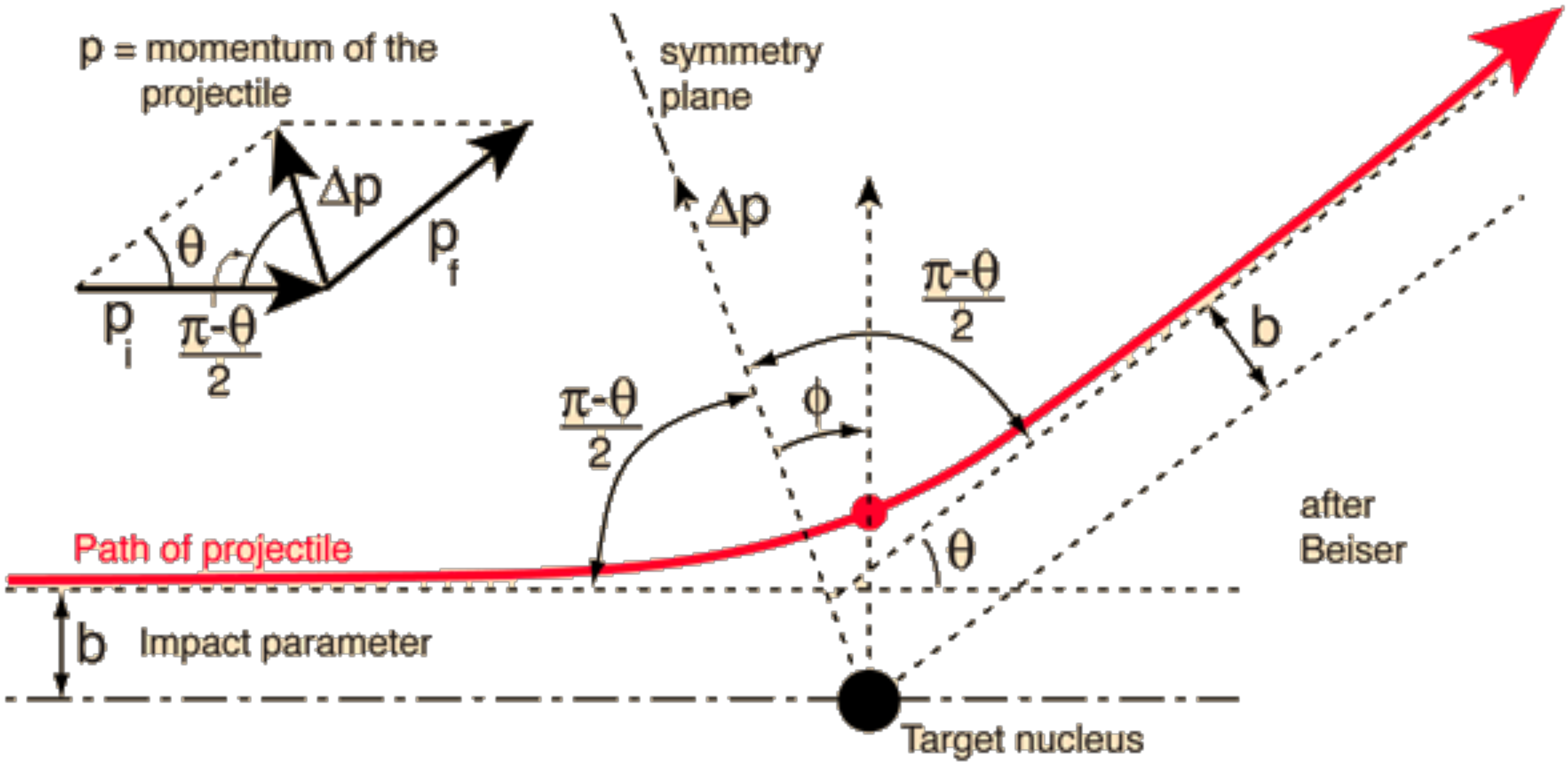
$$\vec{F} = \frac{d\vec{p}}{dt} \Longrightarrow \Delta\vec{p} = \int \vec{F} dt$$

The force is the Coulomb force

$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{Z_1 Z_2 e^2}{r^2}\frac{\vec{r}}{r}$$

Before we start integrating let us note that the trajectories are symmetric with respect to the line defined by the distance of the closest approach

Trajectories are symmetric with respect to angle ϕ



Rutherford Scattering Formula

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The symmetry with respect to the line at φ = 0 implies

$$\Delta\vec{p} = \int \vec{F} dt \Longrightarrow \Delta p = \int F \cos\phi dt$$

So,

$$\Delta p = \frac{Z_1 Z_2 e^2}{4\pi\varepsilon_0} \int \frac{1}{r^2} \cos\phi dt$$

This integral can be carried over with a help of conservation of angular momentum.

The angular momentum is

$$\vec{L} = \vec{r} \times \vec{p} = m\vec{r} \times \vec{v} = m\vec{r} \times \left(\frac{d\vec{r}}{dt} + r\frac{d\vec{\phi}}{dt} \right) = mr\vec{r} \times \frac{d\vec{\phi}}{dt}$$

The magnitude of angular momentum

$$L = |\vec{L}| = mr^2 \frac{d\vec{\phi}}{dt}$$

From the initial condition

$$L = mv_0 b$$

Since the angular momentum is conserved

$$mr^2 \frac{d\phi}{dt} = mv_0 b$$

$$\frac{dt}{r^2} = \frac{d\phi}{v_0 b}$$

Rutherford Scattering Formula

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Thus the change of momentum

$$\Delta p = \frac{Z_1 Z_2 e^2}{4\pi\varepsilon_0}\int \frac{dt}{r^2}\cos\phi = \frac{Z_1 Z_2 e^2}{4\pi\varepsilon_0}\int \frac{d\phi}{v_0 b}\cos\phi$$

$$= \frac{Z_1 Z_2 e^2}{4\pi\varepsilon_0}\frac{1}{v_0 b}\int_{\phi_<}^{\phi_>} d\phi\cos\phi$$

The limits for integration are

$$\phi_> = \frac{1}{2}(\pi - \theta)$$

$$\phi_< = -\frac{1}{2}(\pi - \theta)$$

Rutherford Scattering Formula

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The integral is

$$\Delta p = \frac{Z_1 Z_2 e^2}{4\pi\varepsilon_0}\frac{1}{v_0 b}\int_{\phi_<}^{\phi_>} d\phi\cos\phi = \frac{Z_1 Z_2 e^2}{4\pi\varepsilon_0}\frac{1}{v_0 b}(\sin\phi_> - \sin\phi_<)$$

$$= \frac{Z_1 Z_2 e^2}{4\pi\varepsilon_0}\frac{2}{v_0 b}\cos\frac{\theta}{2}$$

Since

$$\Delta p = 2p\sin(\theta/2)$$

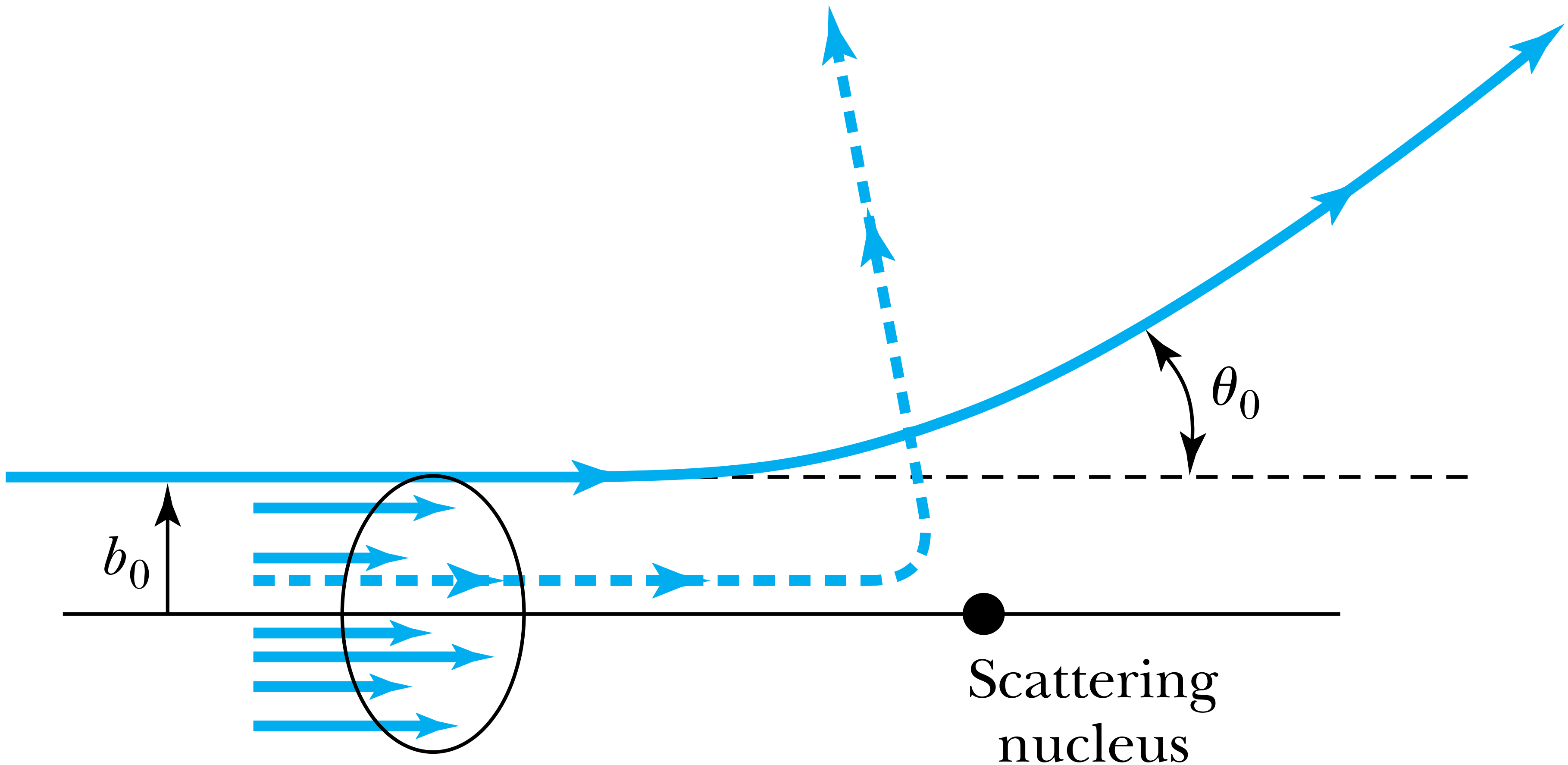
The impact parameter is expressed as

$$b = \frac{Z_1 Z_2 e^2}{4\pi\varepsilon_0}\frac{1}{p v_0}\frac{1}{\tan(\theta/2)}$$

$$= \frac{Z_1 Z_2 e^2}{4\pi\varepsilon_0}\frac{1}{2E}\frac{1}{\tan(\theta/2)}$$

with E being the initial kinetic energy for the projectile

The cross section



The cross section

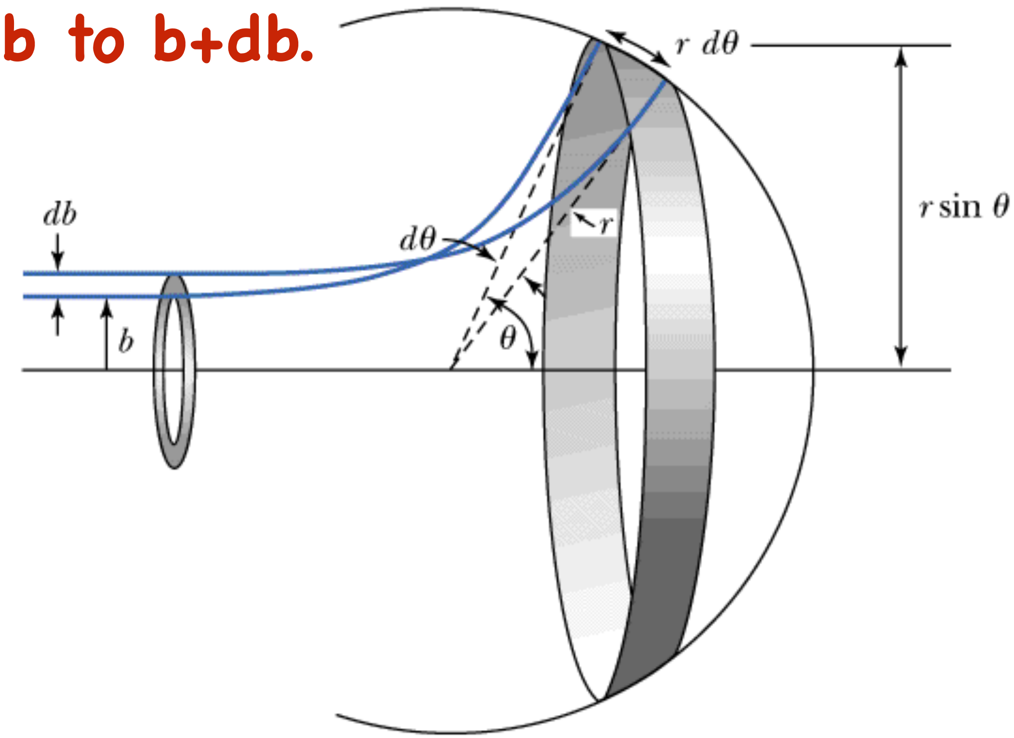
All the particles with impact parameters less than b_0 will be scattered at angles greater than θ_0. Any particle with an impact parameter inside the area of the circle[2] of area πb^2_0 (radius b_0) will be similarly scattered. For the case of Coulomb scattering, we denote the cross section by the symbol σ, where

$$\sigma = \pi b^2$$

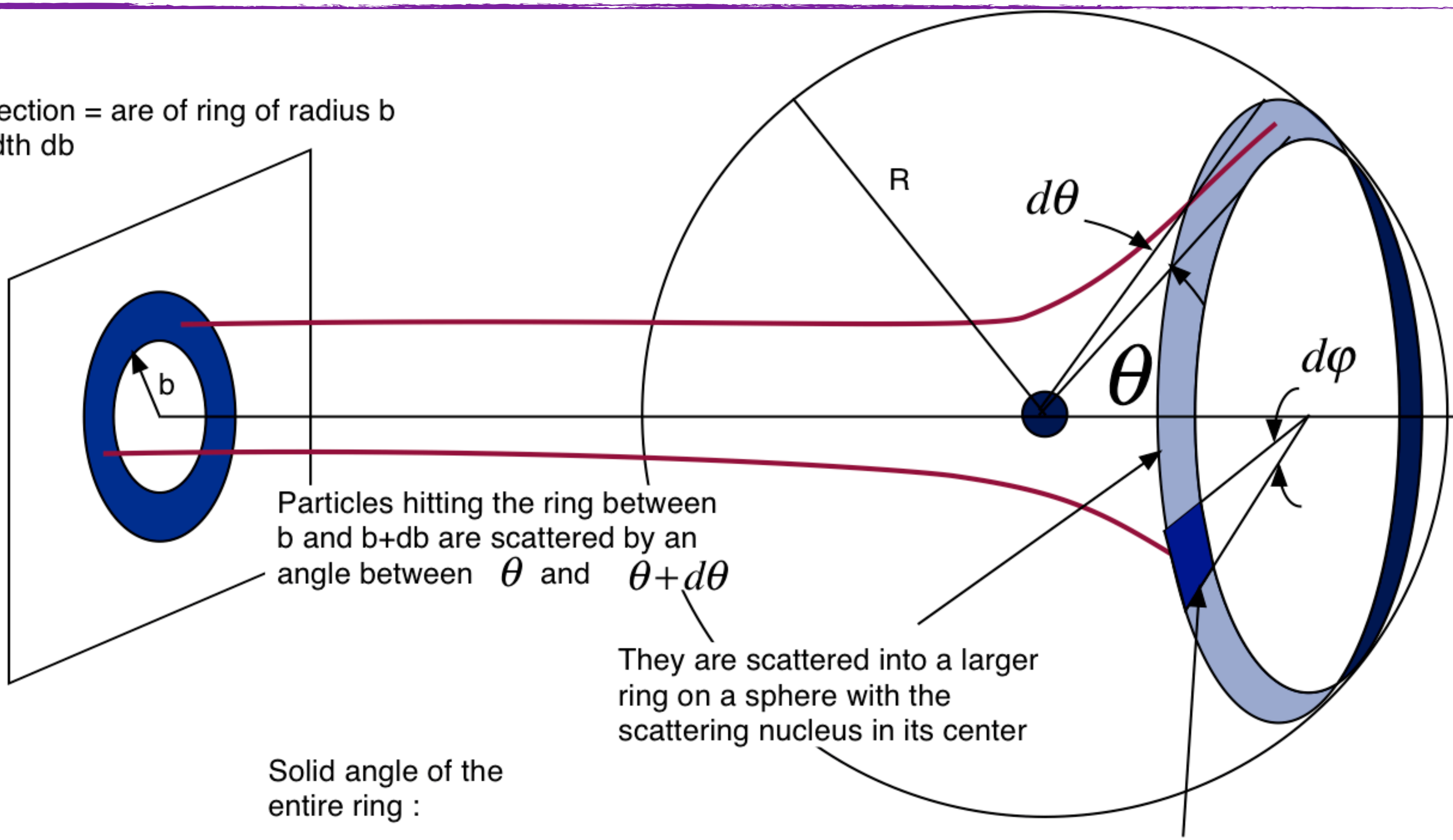
is the cross section for scattering through an angle θ or more. The cross section σ is related to the probability for a particle being scattered by a nucleus.

The cross section

In an actual experiment, we cannot measure the number of scattered particles arriving at a particular angle θ, but rather we have to consider the finite range of angles between θ and θ+dθ; these correspond to impact parameters in the range b to b+db.



The cross section



Solid angle of the entire ring :

$$d\Omega = \frac{2\pi R\sin(\theta)Rd\theta}{R^2} = 2\pi\sin(\theta)d\theta$$

solid angle of small area:

$$d\Omega = \frac{d\varphi R\sin(\theta)Rd\theta}{R^2} = \sin(\theta)d\theta d\varphi$$

The cross section

Since particles from the ring defined by the impact parameters b and b+db scatter between angles θ and θ+dθ the cross section for scattering into the angle θ (called the differential cross section) is

$$\frac{d\sigma}{d\Omega} = \frac{2\pi b db}{2\pi \sin\theta d\theta} = \frac{b}{\sin\theta}\frac{db}{d\theta}$$

The relationship between b and θ for the Rutherford scattering yields

$$\frac{d\sigma}{d\Omega} = \left(\frac{Z_1 Z_2 e^2}{4\pi\varepsilon_0}\frac{1}{4E}\right)^2 \frac{1}{\sin^4(\theta/2)}$$

The cross section

If N incident particles strike a foil of thickness t cotaining n scattering centers per unit volume, the average number dN of particles Ωscattered into the solid angle dΩ around Ω is given by

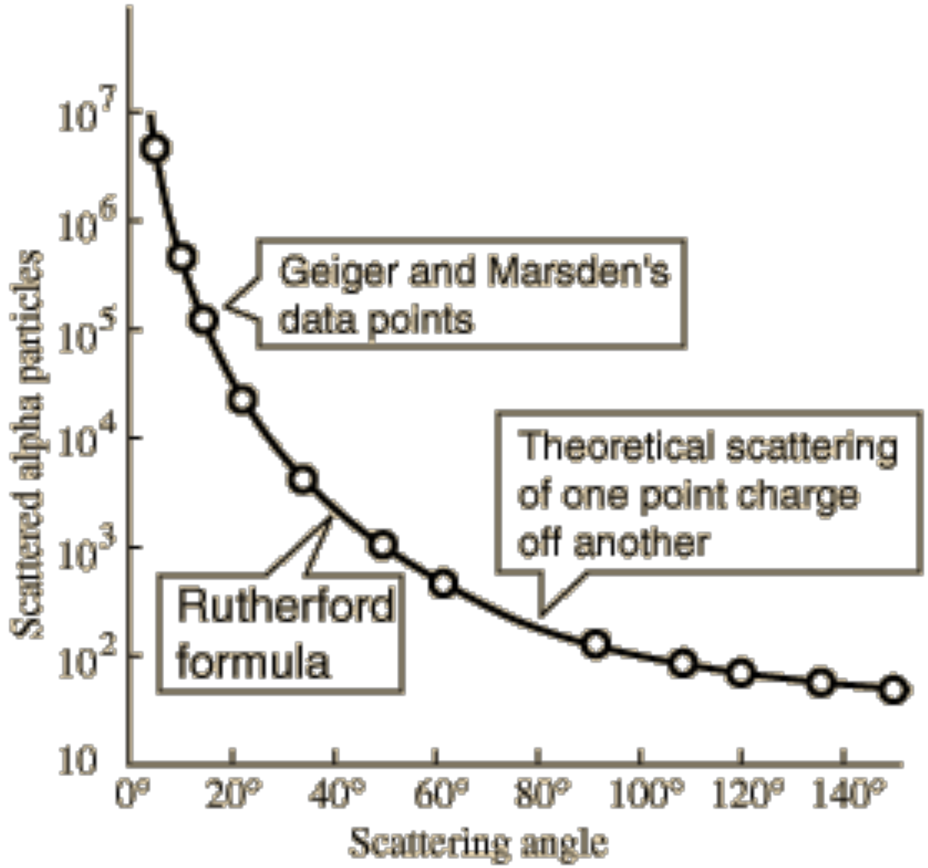
$$dN = Nnt\frac{d\sigma}{d\Omega}d\Omega$$

Therefore,

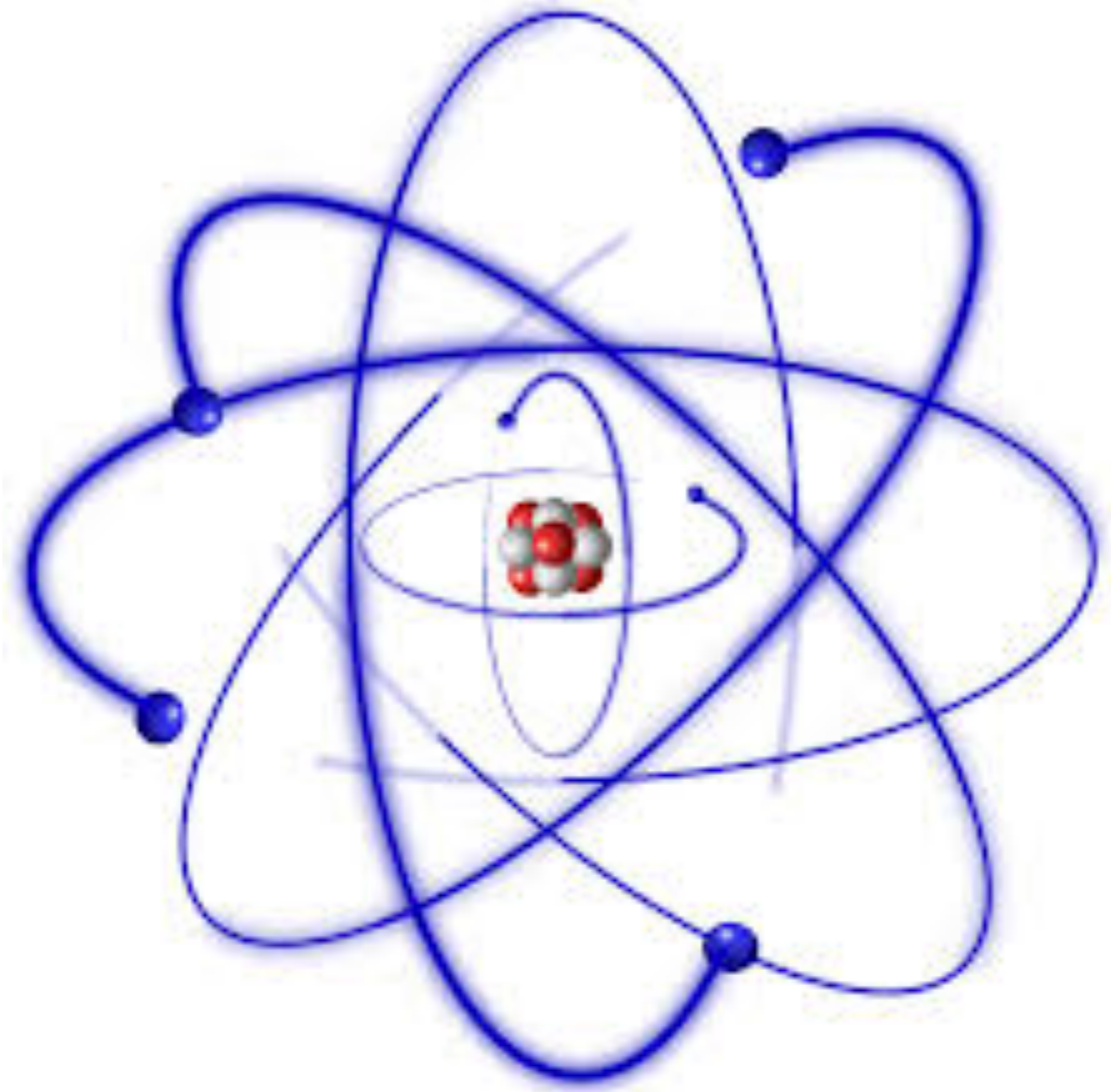
$$\frac{dN}{N} = nt\left(\frac{Z_1 Z_2 e^2}{16\pi\varepsilon_0 E}\right)^2 \frac{1}{\sin^4(\theta/2)}d\Omega$$

This is the Rutherford result explaining the Geiger-Marsden experiment

Number of particles scattered at a given angle in Rutherford scattering is calculable and well understood, since it is defined by the well understood electromagnetic force.



Rutherford planet model



The difference between Thomson

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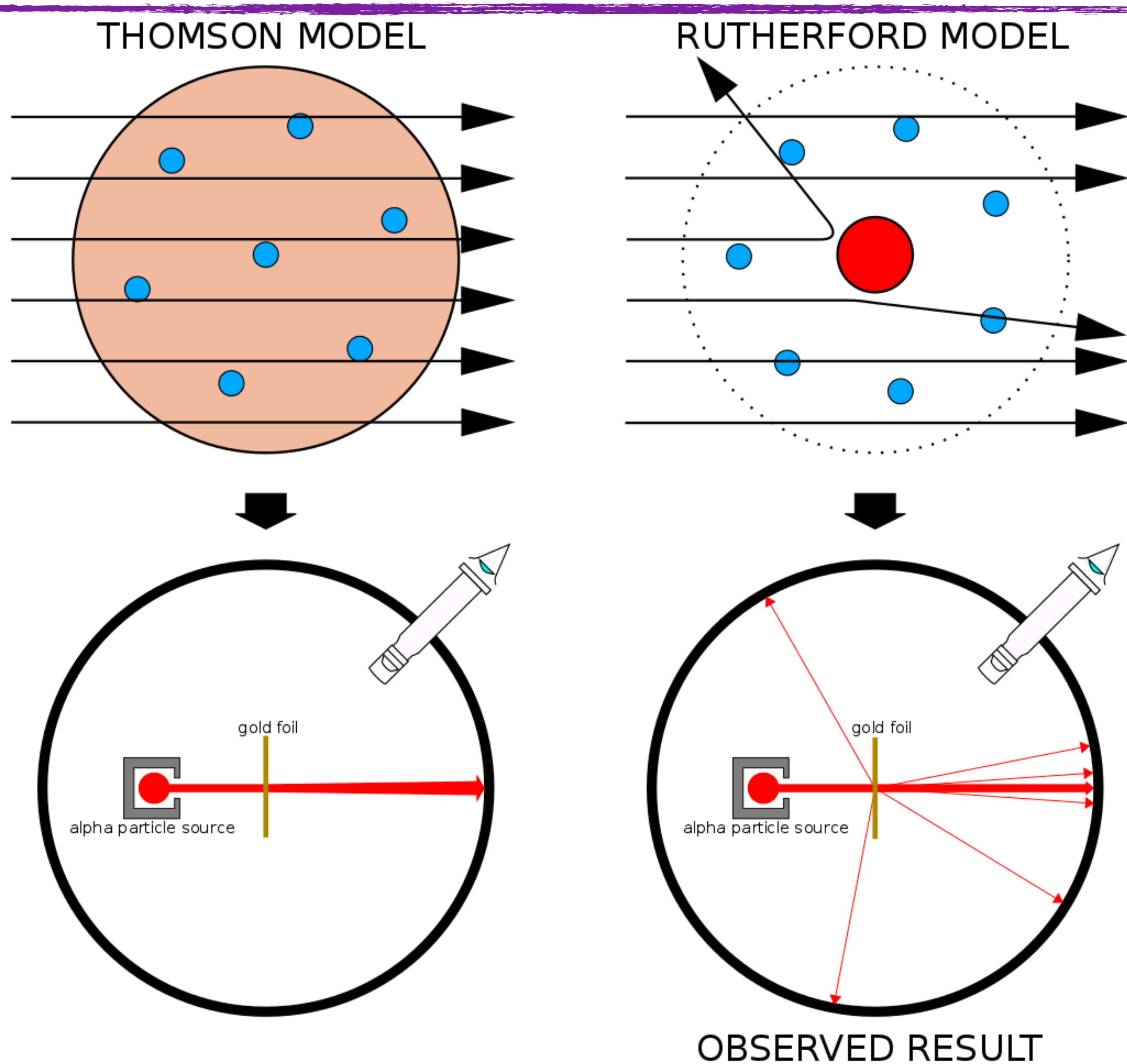

The key points

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Impact parameter: b

$$\frac{d\sigma}{d\Omega} = \left(\frac{Z_1 Z_2 e^2}{4\pi\varepsilon_0}\frac{1}{4E}\right)^2 \frac{1}{\sin^4(\theta/2)}$$

Scattering angle: θ

Differential cross-section: the number of particles scattered into an element of solid angle dΩ in the direction θ per unit area (unit 1 barn=10^{-28} m^2)

The important quantities in Rutherford formula:

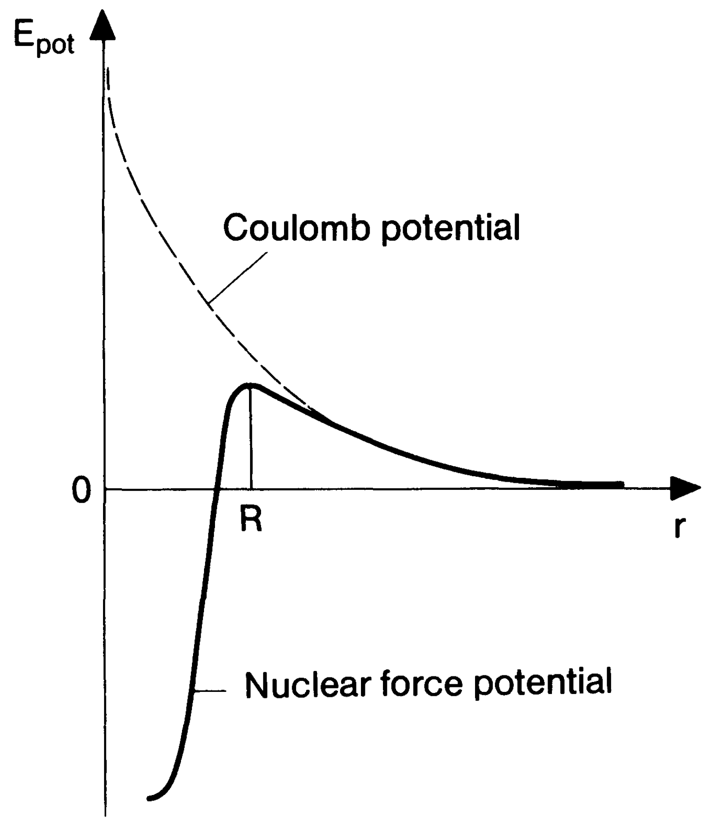
1. impact parameter
2. Scattering angle.
3. charges
4. Initial kinetic energy

What is Meaning by Nuclear Radius?

The nuclear radius is defined as the distance at which the effect of the nuclear potential is comparable to that of the Coulomb potential

$$E = \frac{Z_1 Z_2 e^2}{4\pi\varepsilon_0}\frac{1}{r}$$

$$r_{\min} = \frac{Z_1 Z_2 e^2}{4\pi\varepsilon_0}\frac{1}{E}$$



Empirical nuclear radius

$$R = 1.2A^{1/3}\ \mathrm{fm}$$

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The Physics of Atoms and Quanta

2.2, 2.4, 4.3, 4.4, 4.5, 4.8

Exercise 1

Magnesium occurs in nature in three isotopic forms:

	Relative mass
^{24}Mg (78.70% abundance)	23.985 amu
^{25}Mg (10.13% abundance)	24.986 amu
^{26}Mg (11.17% abundance)	25.983 amu

Calculate the atomic mass of Magnesium from these data.

Exercise 1

Magnesium occurs in nature in three isotopic forms:

	Relative mass
^{24}Mg (78.70% abundance)	23.985 amu
^{25}Mg (10.13% abundance)	24.986 amu
^{26}Mg (11.17% abundance)	25.983 amu

Calculate the atomic mass of Magnesium from these data.

Solutions:

$$M_{Mg} = 0.7870*23.985 + 0.1013*24.986 + 0.1117*25.983$$
$$= 24.31 \text{ amu}$$

The atomic mass of Magnesium is 24.31 amu

Exercise 2

The density of gold (^{197}Au) is 19.3 g/cm^3. How many gold atoms are present in a piece of gold whose volume is 3.22 cm^3 ?

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Solutions:

One gold $m_{Au}=197*1.66*10^{-27}$ kg$=3.27*10^{-22}$ g

One piece of gold $M_{Au}=19.3*3.22=62.15$ g

$N_{Au}=62.15/(3.27*10^{-22})=1.90*10^{23}$

There are $1.90*10^{23}$ gold atoms in a piece of gold whose volume is 3.22 cm^3 .

Exercise 3

Calculate the impact parameter for scattering a 7.7 MeV α particle from gold at an angle of (a) 1° and (b) 90°.

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Solutions:

The α particle and gold nucleus have Z1 = 2 and Z2 = 79, respectively. For θ=1°,

$$b = \frac{Z_1 Z_2 e^2}{4\pi\varepsilon_0}\frac{1}{2E}\frac{1}{\tan(\theta/2)} \qquad \frac{e^2}{4\pi\varepsilon_0} = 1.44\times10^{-9}\ \mathrm{eV\ m}$$

$$= \frac{2\times79\times1.44\times10^{-9}}{2\times7.7\times10^6}\frac{1}{\tan(0.5\pi/180)} = 1.69\times10^{-12}\ \mathrm{m}$$

for θ=90°,

$$b = \frac{Z_1 Z_2 e^2}{4\pi\varepsilon_0}\frac{1}{2E}\frac{1}{\tan(\theta/2)}$$

$$= \frac{2\times79\times1.44\times10^{-9}}{2\times7.7\times10^6}\frac{1}{\tan(\pi/4)} = 1.48\times10^{-14}\ \mathrm{m}$$

Exercise 4

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Rutherford found deviations from his scattering formula at backward angles when he scattered 7.7 MeV α particles (Z_1=2) on aluminum (Z_2=13). He suspected this was because the α particle might be affected by approaching the nucleus so closely. Estimate the size of the nucleus based on these data.

Exercise 4

Rutherford found deviations from his scattering formula at backward angles when he scattered 7.7 MeV α particles (Z_1=2) on aluminum (Z_2=13). He suspected this was because the α particle might be affected by approaching the nucleus so closely. Estimate the size of the nucleus based on these data.

Solutions:

$$r_{\min} = \frac{Z_1 Z_2 e^2}{4\pi\varepsilon_0}\frac{1}{E}$$

$$= \frac{2 \times 13 \times 1.44 \times 10^{-9}}{7.7 \times 10^6} = 4.9 \text{ fm}$$

We find the sum of the ^{4}He and aluminum nuclear radii to be about $5*10^{-15}$ m.