



# Quantum Mechanics

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参考书:

曾谨言 《量子力学》 卷I (第四版)

苏汝铿 《量子力学》

周世勋 《量子力学教程》

Landau, L. D. & Lifshitz, M. E., *Quantum Mechanics*

Greiner, W., *Quantum Mechanics*

*Sakurai, J. J., Modern Quantum Mechanics*



# Chapter 1: birth of quantum mechanics

Planck, Bohr, Einstein, Heisenberg, Born, Pauli, De Broglie, Schrodinger, Dirac....

## *What is Quantum Mechanics?*

QM is the theory of the behavior of very small objects  
(e.g. molecules, atoms, nuclei, elementary particles, etc.)

## *Why Quantum Mechanics?*

One of the essential differences between classical and quantum mechanics is that physical variables that can take on continuous values in classical mechanics (e.g. energy, angular momentum) can only take on discrete (or quantized) values in quantum mechanics (e.g. the energy levels of electrons in atoms, or the spins of elementary particles, etc).

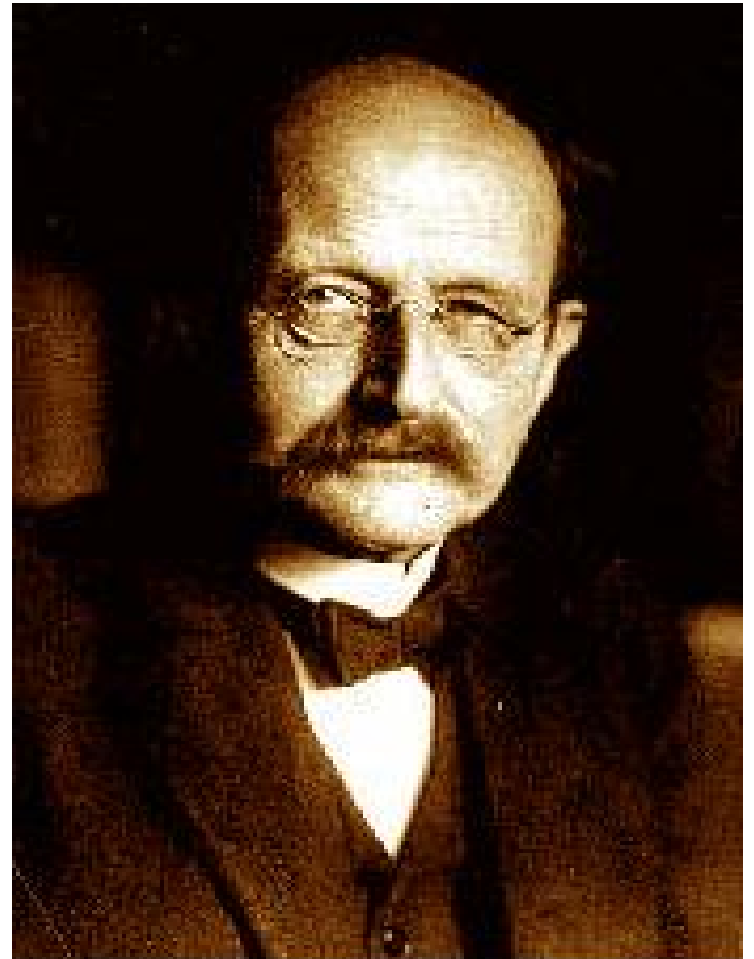
# birthday of quantum mechanics

14 December 1900

Planck (age 42)  
suggests that  
radiation is quantized

$$E = h\nu$$

$$h = 6.626 \times 10^{-34} \text{ J}\cdot\text{s}$$



Max Planck (1858-1947)

Nobel Prize 1918



## success of classical physics

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- ◆ At the turn of the 20<sup>th</sup> Century, it was thought that physics had just about explained all natural phenomena.
- ◆ During the preceding 3 centuries (~1600-1900)
  - Newtonian Mechanics
    - ◆ Forces and motion of Particles, fluids, waves, sound
    - ◆ Universal theory of gravity
  - Maxwell's Theory of Electromagnetism (EM)
    - ◆ Unified electric and magnetic phenomena
    - ◆ Thoroughly explained electric and magnetic behaviour
    - ◆ Predicted existence of electromagnetic waves
  - Thermodynamics
    - ◆ Thermal processes
    - ◆ Kinetic theory of gases and other materials



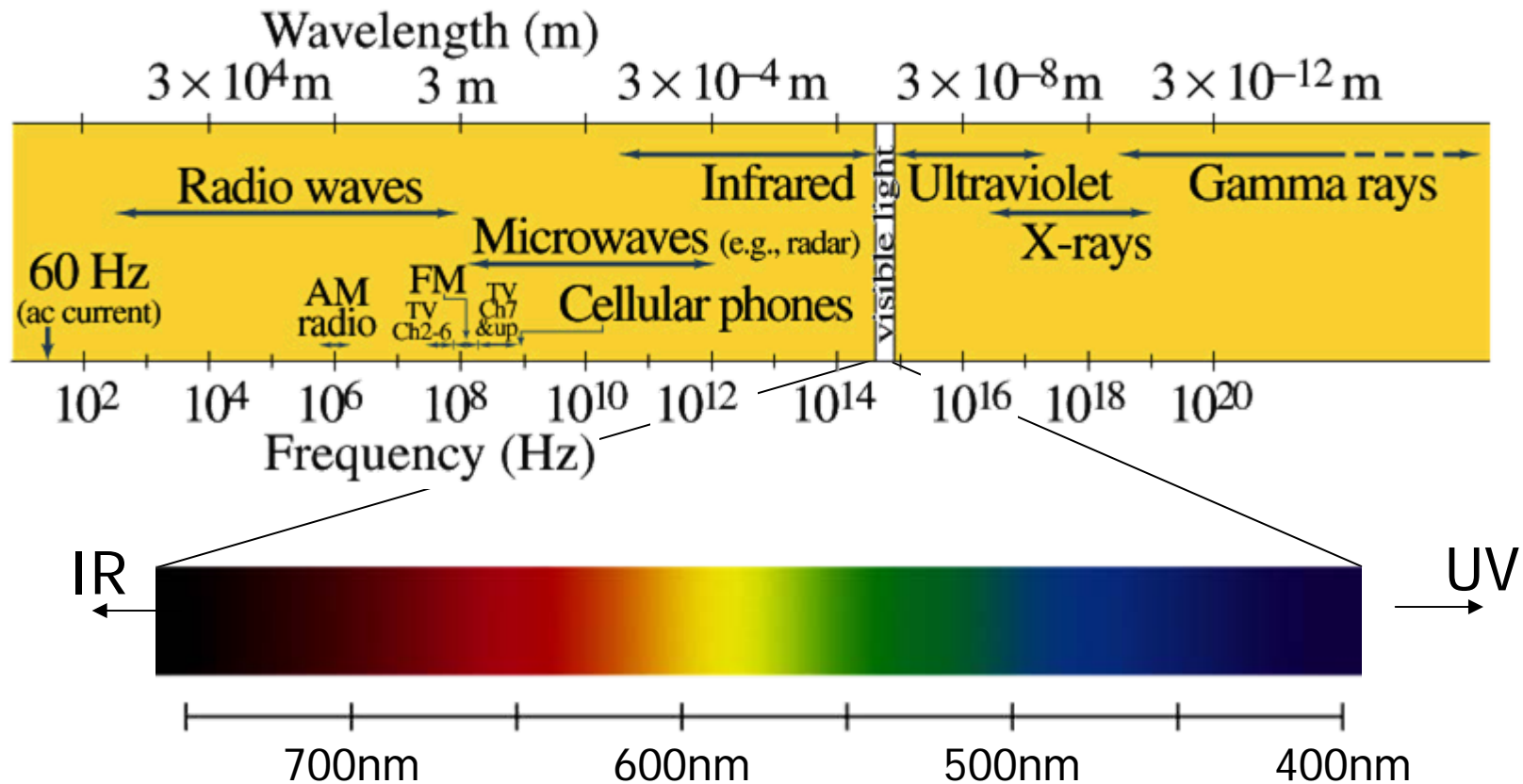
# difficulty of classical physics

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- ◆ Newtonian Mechanics + Maxwell electrodynamics = wrong answer (Maikelson-Morli experiment)  
# resolution: relativity theory
- ◆ Electrodynamics + Statistics = wrong answer (blackbody radiation, photoelectric effect)  
# resolution: quantum mechanics, quantum electrodynamics
- ◆ atomic models and atomic line spectrum (Rutherford experiments, spectral experiments)  
# resolution: quantum mechanics

# blackbody radiation

- Light is an electromagnetic wave (accelerating charges)
- Electromagnetic Spectrum

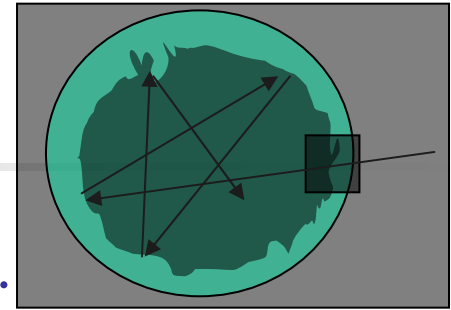


# blackbody radiation

◆ An object at any temperature emits radiation.

◆ A *blackbody* is an object which totally absorbs all radiation that falls on it.

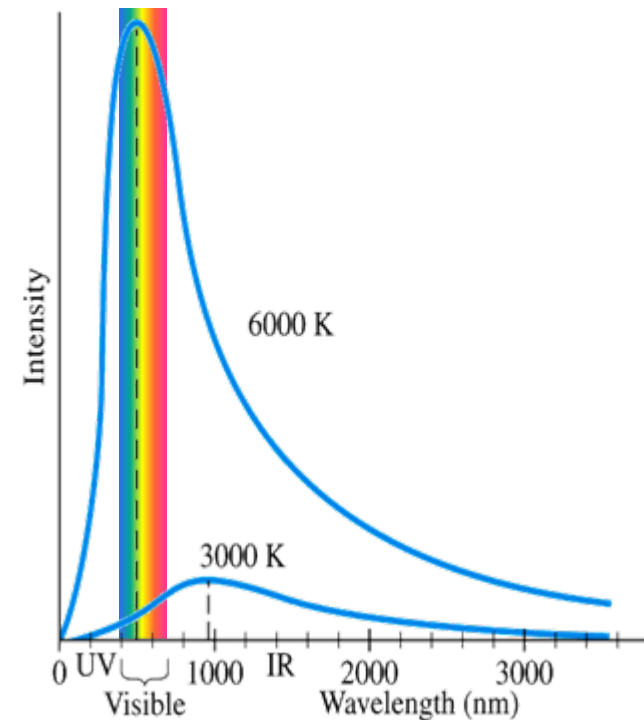
◆ A *blackbody* radiates light over the whole spectrum of frequencies.



The hole is a perfect *blackbody*

## Why physicists were so interested in the "blackbody radiation" ?

Electromagnetic radiation is clearly a subject worth investigating. Blackbody radiation is a specific aspect of electromagnetic radiation. Blackbody has universal spectrum dependent only upon temperature (independent of the details of the radiating object).



# blackbody radiation

- ◆ Total intensity increases with T  
Stefan-Boltzmann law

$$R = \sigma T^4$$

- ◆ Peak wavelength moves to shorter wavelengths with increasing T

- ◆ Wien's Law

$$E_\nu d\nu = c_1 \nu^3 \exp[-c_2 \nu / T] d\nu$$

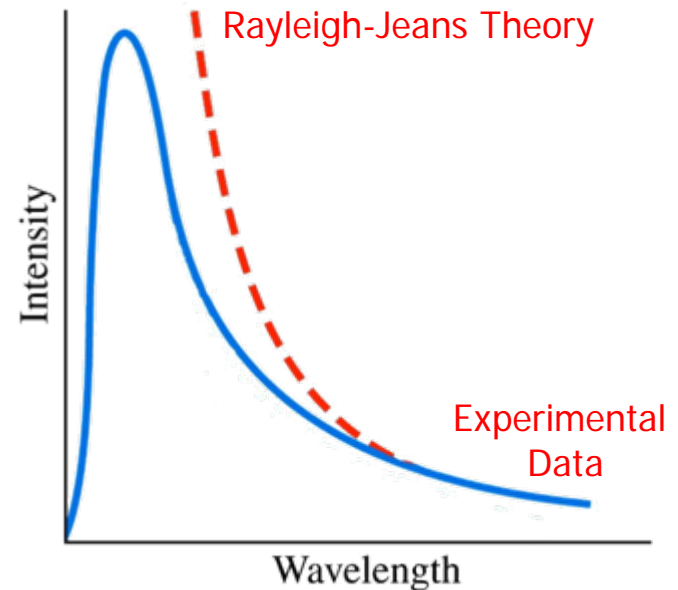
fits data well for short wavelengths

- ◆ Rayleigh-Jeans Law

$$E_\nu d\nu = \frac{8\pi}{c^3} k T \nu^2 d\nu$$

fits data well for long wavelengths

Classical physics prediction *Ultraviolet Catastrophe*







# blackbody radiation

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◆ In 1900 Planck proposed a formula which nicely fit the data

$$E_{\nu}d\nu = \frac{8\pi h}{c^3} \frac{\nu^3}{\exp[h\nu / kT] - 1} d\nu$$

$h = 6.626 \times 10^{-34} \text{ J s}$  Planck's constant

*What's the BIG IDEA here? Planck's Quantum Hypothesis*

Oscillators can only emit energy in units of  $h\nu$ ,  
called "quanta" of energy.  $E = h\nu$

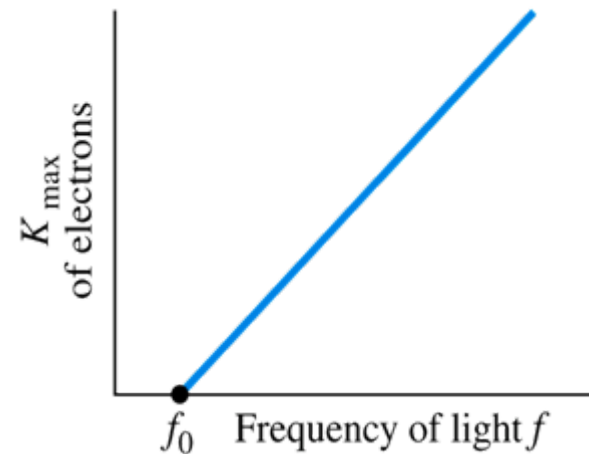
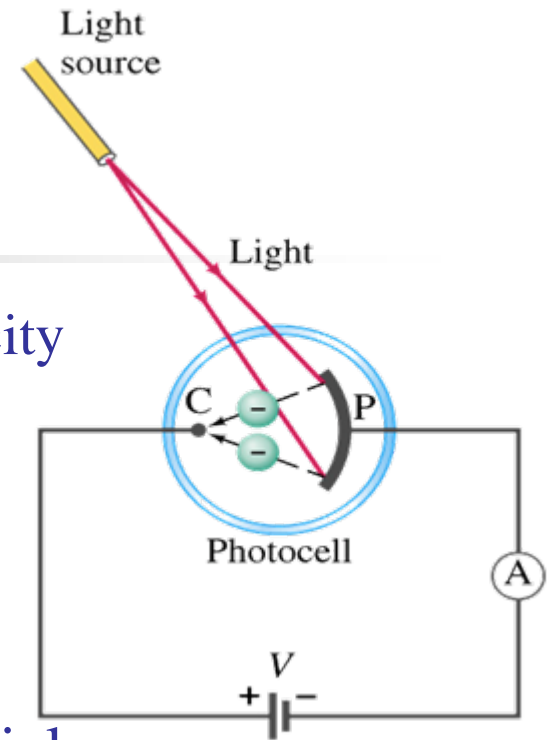
radiation is quantized

# photoelectric effect

Hertz (1888) observed that light can produce electricity

## *Properties of the photoelectric effect*

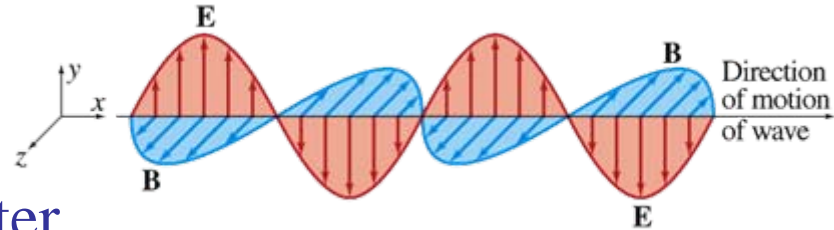
- ◆ Electrons are only emitted above a certain “cut-off” frequency.
- ◆ This frequency is different for different materials.
- ◆ The current increases with light intensity.
- ◆ The maximum energy of the electron is independent of the light intensity.
- ◆ The maximum energy of the electron increases with increasing frequency.
- ◆ Electrons are emitted almost instantaneously.



# photoelectric effect

## *Predictions of Classical Wave Theory*

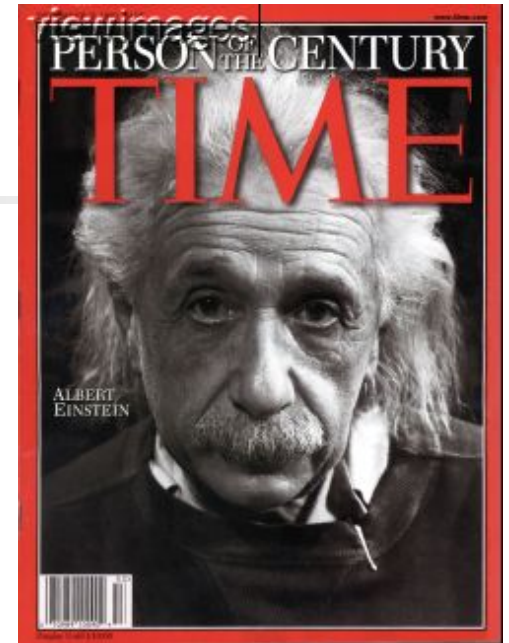
- E&M wave can exert a force on electrons in the metal and eject some of them.
- If the light intensity is increased,
  - Electric field amplitude is greater
  - number of electrons ejected increases
  - kinetic energy of ejected electrons increases ✗
- If the frequency of the light is increased,
  - Nothing should happen.
  - kinetic energy is independent of the frequency ✗
- A time delay should exist before electrons are emitted ✗



# photoelectric effect

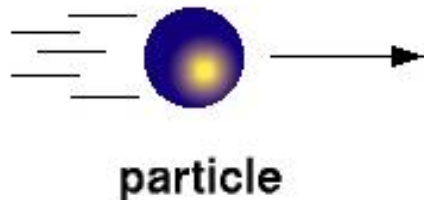
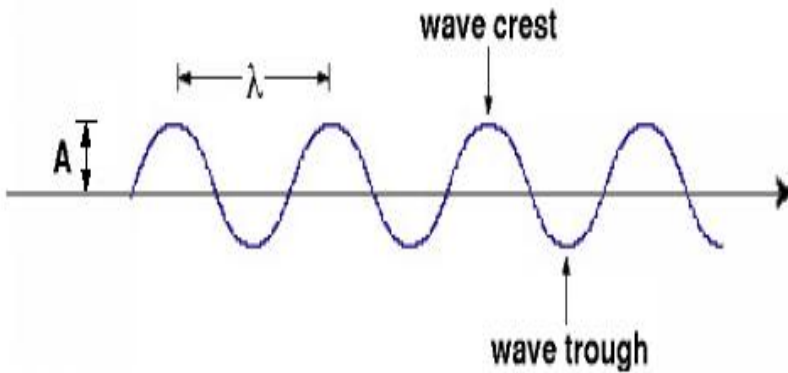
## *Einstein's Theory of Light*

- Albert Einstein (1879-1955) Nobel Prize 1921  
in 1905 Einstein (age 26) proposed *photons*
- Energy is related to frequency and wavelength  $E = h\nu = hc / \lambda$
- The *KE* of an emitted electron is given by  $KE = h\nu - A$   
*A* is the energy required to remove from the material
- If the light frequency is below  $\nu_0$ , no electrons will be emitted  
 $A = h\nu_0$   
 $KE_{\max} = h\nu - h\nu_0$
- More intensity  $\rightarrow$  more photon  $\rightarrow$  more electrons
- Ejection of the first electron should be instantaneous



# wave-particle duality

*In classical physics, wave and particle are different, waves superpose and pass through each other, while particles collide and bounce off each other.*



*superposition  
interference  
diffraction*

...

$X, M,$   
 $P, E,$

...

# wave-particle duality

*Is light a wave or a particle?*

## □ Wave

- Electric and Magnetic fields act like waves
- Superposition, Interference and Diffraction

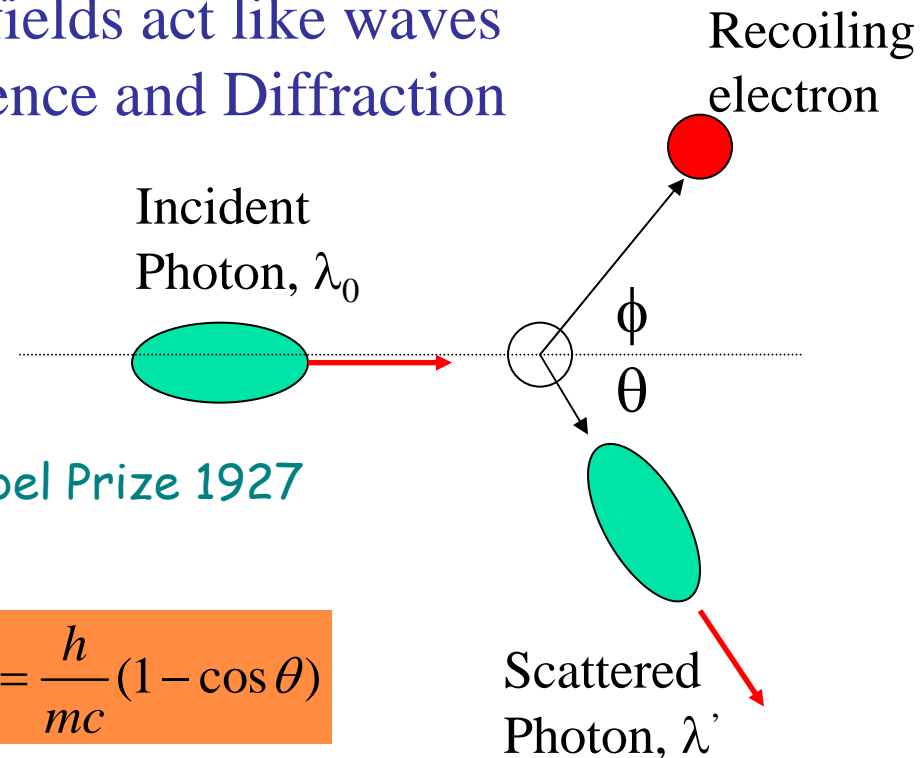
## □ Particle

- Photons
- Photoelectric effect
- Compton scattering Nobel Prize 1927

photons have momentum !

$$p = \frac{E}{c} = \frac{h\nu}{c} = \frac{h}{\lambda} \quad \longrightarrow \quad \lambda' - \lambda_0 = \frac{h}{mc} (1 - \cos \theta)$$

sometimes particle sometimes wave

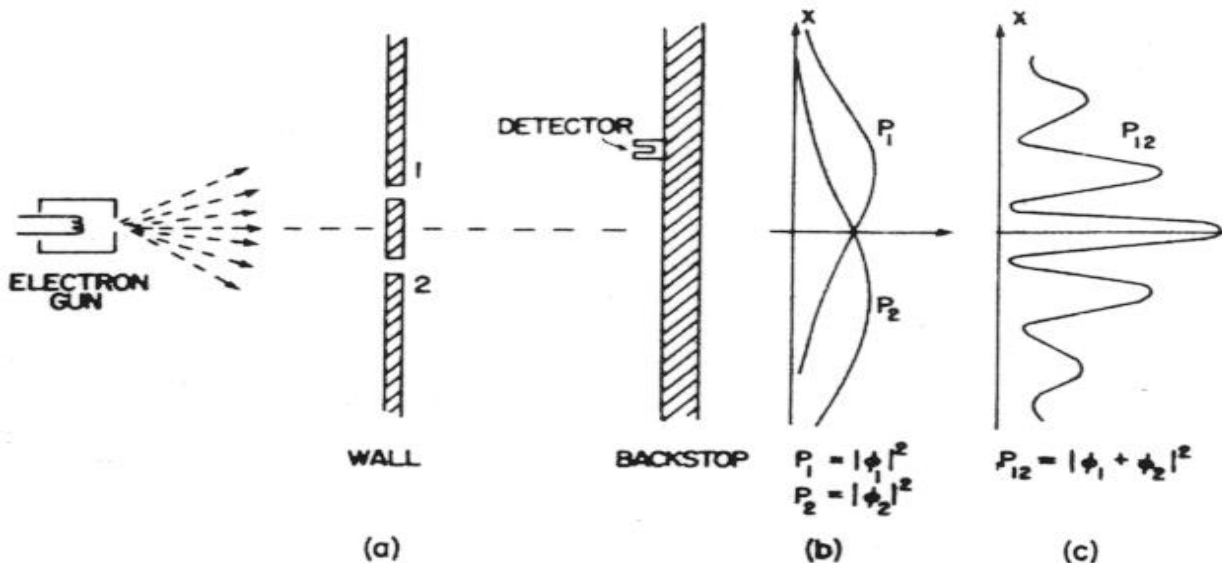


# wave-particle duality

*In 1923 Louis de Broglie (age 31) postulated matter has wave properties Nobel Prize 1929*

*Is an electron a particle or a wave?*

- ❑ Particle, definitely particle.  $m$ ,  $q$ ,  $E$ ...
- ❑ Wave. electron interference, electron diffraction...



$$\nu = \frac{E}{h}, \quad \lambda = \frac{h}{p}$$

# wave-particle duality

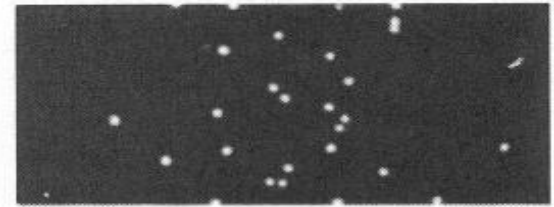
*why we do not see its wave properties?*

wavelength is very small

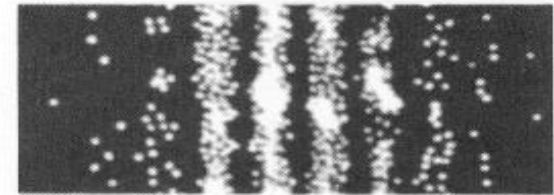
$E = 20 \text{ eV}$ ,  $\lambda = 0.27 \text{ nm}$   
same size as crystal lattice

sometimes particle sometimes wave

*In quantum theory, the distinction  
between waves and particles is blurred*



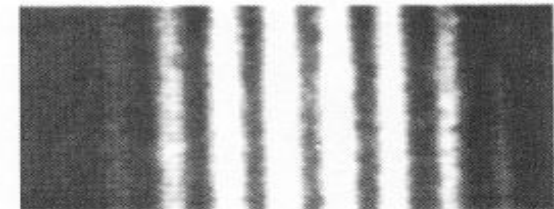
(a) After 28 electrons



(b) After 1000 electrons



(c) After 10,000 electrons

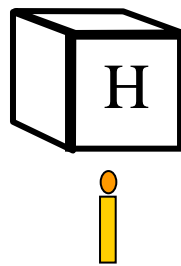


(d) Two-slit electron pattern



# atomic line spectrum

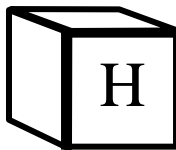
In addition to the continuous blackbody spectrum, elements emit a discrete set of wavelengths (bright lines).



*Hydrogen emission spectrum*



cool gasses absorb certain wavelengths (dark lines).



*Hydrogen absorption spectrum*



white light  
(all colors)

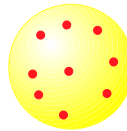
Balmer series in 1885, Lyman series...

# Bohr's model of the atom

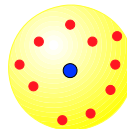
In 1897 the electron was discovered  
by Thomson (Nobel Prize 1906)

*what is the structure of the atom ?*

➤ Thomson's model  
"plum pudding" model **x**



➤ Rutherford's model  
"nuclear atom" model **v**



*Why don't electrons fall into the nucleus?  
Nuclear atom not classically stable !*

***need quantum theory !***

➤ Bohr's model



Niels Bohr (1885-1962)  
Nobel Prize 1922

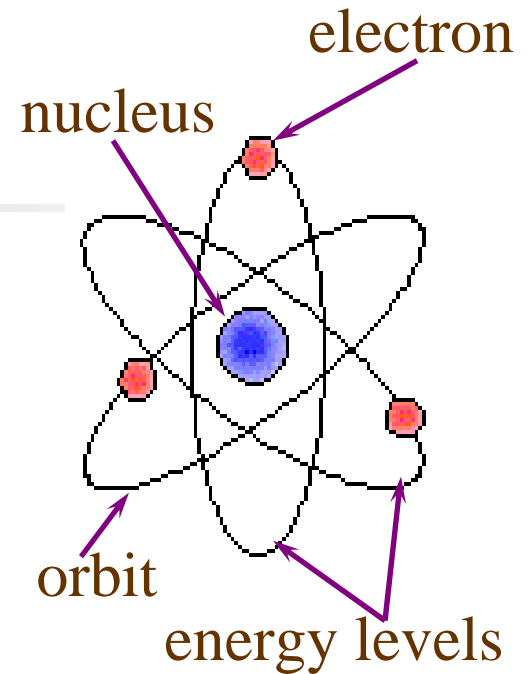
# Bohr's model of the atom

1913 Bohr (age 28) constructs a theory of atoms

1921 Bohr Institute opened in Copenhagen  
(Pauli, Heisenberg, Dirac, ...)

## *assumptions and limitations of Bohr model*

- electrons follow circular orbits.
- electrons could have only certain size orbits (quantum condition), equivalent to quantized angular momentum.
- electrons are allowed to higher orbit with an input energy.
- gets correct energy levels and radius of hydrogen atom.
- fails for multi-electron atoms (even He)...
- Bohr model is semi-classical (violate uncertainty principle).



# birth of quantum mechanics

Quantum mechanics arose from two independent schemes which were introduced by a pair of young remarkable physicists: Austrian, Erwin Schrödinger and German, Werner Heisenberg.

1926 Schrödinger (age 39)



Erwin Schrödinger (1887-1961)  
Nobel Prize 1933

1925 Heisenberg (age 23)



Werner Heisenberg (1901-1976)  
Nobel Prize 1932

# birth of quantum mechanics



- wave function formulation of quantum mechanics
- matrix formulation of quantum mechanics
- path integral formulation of quantum mechanics

Max Born (1882-1970)

Nobel Prize 1954

Schrödinger and Heisenberg were not too fond of each other's competing works.

*"I knew of [Heisenberg's] theory, of course, but I felt discouraged, not to say repelled, by the methods of ... which appeared difficult to me, and by the lack of visualizability. "*

*- Erwin Schrodinger (1926) -*

*"The more I think of the physical part of the Schrödinger theory, the more detestable I find it. What Schrödinger writes about visualization makes scarcely any sense, in other words I think it is shit ..."*

*- Werner Heisenberg (8 June 1926) -*



# birth of quantum mechanics

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1926 Erwin Schrödinger in Austria  
Carl Eckert (age 24) in America

Proved: **wave mechanics = matrix mechanics**

(Schrödinger and Heisenberg theories equivalent mathematically)

Schrödinger's wave mechanics eventually became the method of choice, because it is less abstract and easier to understand than Heisenberg's matrix mechanics

Neumann (mathematician) invented operator theory  
Largely because of his work (publish his book in 1932),  
quantum physics and operator theory can be viewed as  
two aspects of the same subject.

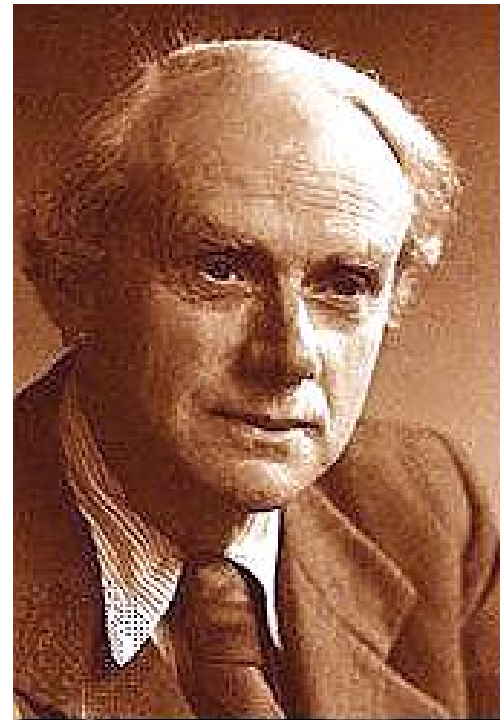
# birth of quantum mechanics

1925 Pauli (age 25)  
Pauli exclusion principle



Wolfgang Pauli (1900-1958)  
Nobel Prize 1945

1928 Dirac (age 26)  
Dirac equation (quantum+relativity)



Paul Dirac (1902-1984)  
Nobel Prize 1933



# 1927 Solvay Conference



*Held in Belgium,  
the conference was attended by the world's most notable physicists  
to discuss the newly formulated quantum theory.*





# applications of quantum mechanics

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It was applied to atoms, molecules, and solids.

It solved with ease the problem of helium

It was used to explain chemical bonding

It resolved various questions: structure of stars,  
nature of superconductors,  
:

Even today it is being applied to new problems.

**Quantum mechanics has been tremendously successful !**



# applications of quantum mechanics

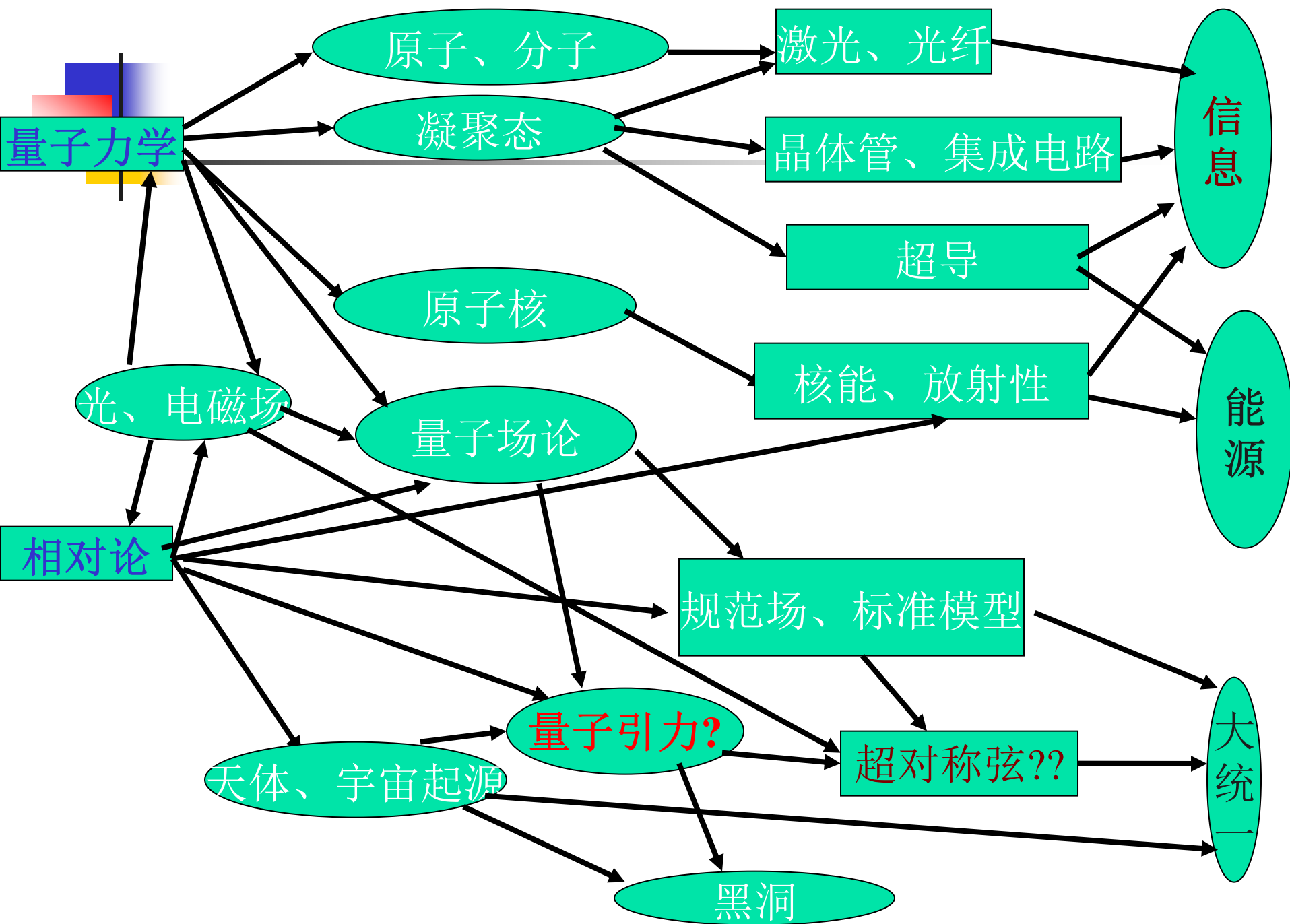
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The applications of quantum mechanics are myriad.

Quantum mechanics underlies all chemical and biochemical reactions, the design of drugs and of alloys, and the generation of medical X-rays.

It is essential to the laser, to the transistor, and thus to computers and mobile phones and many other things.

Light Amplification by Stimulated Emission of Radiation = LASER





## Relation between classical and quantum mechanics

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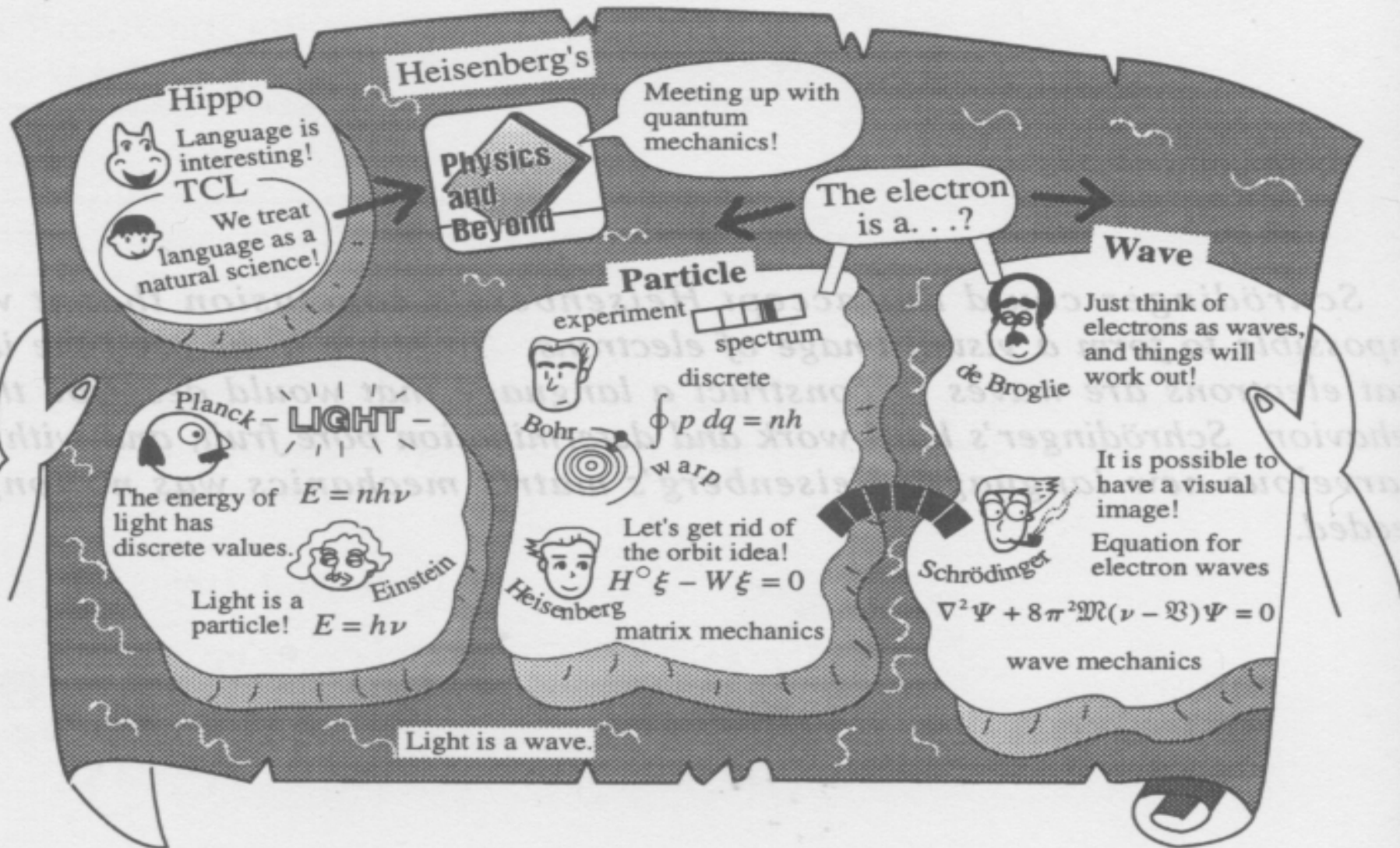
The real world is neither entirely classical nor quantum.

On a “large” scale, the world seems to behave according to the classical theories.

At a “smaller” scale, it starts to follow a different set of rules summarized in quantum mechanics.

Quantum mechanics gave a strange and, in many ways, philosophically unsatisfying view of the world.

# Please enjoy quantum mechanics !



# The size of the things

## Instruments

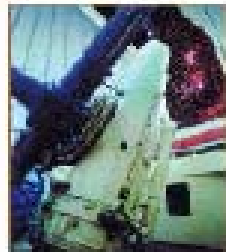
## Observables



Accelerators  
LHC, LEP



(Particle beams)  
Electron  
Microscope  
Microscope



Telescope

Radio  
Telescope

$10^{-34}$

$10^{-30}$

$10^{-26}$

$10^{-22}$

$10^{-18}$

$10^{-14}$

$10^{-10}$

$10^{-6}$

1m

$10^6$

$10^{10}$

$10^{14}$

$10^{18}$

$10^{22}$

$10^{26}$

SUSY particle?

Higgs? (range of  
nuclear force)

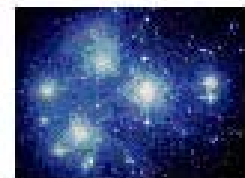
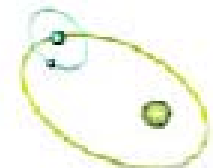
Z/W (range of  
weak force)

Proton  
Nuclei

Atom

Virus

Cell



Earth radius  
Earth to Sun

Galaxies  
Radius of observable  
Universe

## E. Fermi (1901-1954) "The Future of Nuclear Physics" January 10, 1952

Toward the end of his life, Fermi questioned his faith in society at large to make wise choices about nuclear technology.

**He said** (<http://en.wikipedia.org/wiki/Fermi>)

"Some of you may ask, what is the good of working so hard merely to collect a few facts which will bring no pleasure except to a few long-haired professors who love to collect such things and will be of no use to anybody because only few specialists at best will be able to understand them?

In answer to such question[s] I may venture a fairly safe prediction.

History of science and technology has consistently taught us that scientific advances in basic understanding have sooner or later led to technical and industrial applications that have revolutionized our way of life. It seems to me improbable that this effort to get at the structure of matter should be an exception to this rule. What is less certain, and what we all fervently hope, is that man will soon grow sufficiently adult to make good use of the powers that he acquires over nature."